

FEAMAC/CARES Software Coupling Development Effort for CMC Stochastic- Strength-Based Damage Simulation

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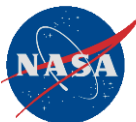
Multiscale & Multiphysics Branch

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Daytona Beach, Florida

CARES: Ceramics Analysis and Reliability Evaluation of Structures
MAC/GMC: Micromechanics Analysis Code/ Generalized Method of Cells

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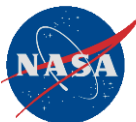
Scope and Technical Challenge

- Predict the strength and service life of ceramic & composite structures

CMC - Ceramic *M*atrix *C*omposites & **PMC** - Polymer *M*atrix *C*omposites

- **Need to account for:**

- Wide variability in the strength of individual components (probabilistic/stochastic strength)
- How strength changes with different types of loading (strength vs: multiaxial loading) and size of the structure (size-effect)
- How strength degrades with time and fluctuating load
- How strength/damage response of monolithic, anisotropic and composite material (architectures) differ



Approach / Outline

1. Overview: Describe the MAC and CARES codes

- **MAC/GMC:** composite micromechanics model
- **CARES Unit Sphere:** multiaxial stochastic strength model
(*isotropy & anisotropy*)

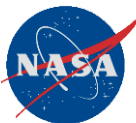
2. Applying CARES to the MAC code to simulate stochastic damage progression in a ceramic matrix composite (CMC)

- ❖ **Cellular Automaton:** Encouraging failure of adjacent elements - mimics crack-like growth
- ❖ Visualization of element-by-element failure propagation for fiber, matrix, and interface

– **Status & Capability:** Current progress of code integration effort

➤ **Examples:**

- (1) Stress-strain response of a SiC-RBSN laminate (circa 1990)
- (2) Time-dependent degradation notional example



MAC/GMC Methodology: Generalized Method of Cells (GMC) & High-Fidelity Generalized Method of Cells (HFGMC)

❖ *Micromechanics links the size scales & provides the composite response based on the composite constituent materials*

■ **FEAMAC:** MAC/GMC embedded in FEA as constitutive material

GMC (1990s)

- 1st order displacement field in subcells
- Stresses and strains piecewise constant
- Number of linear algebraic equations function of number of subcells
- Local inelasticity/damage
- *No shear coupling*
- No “subcell mesh” sensitivity

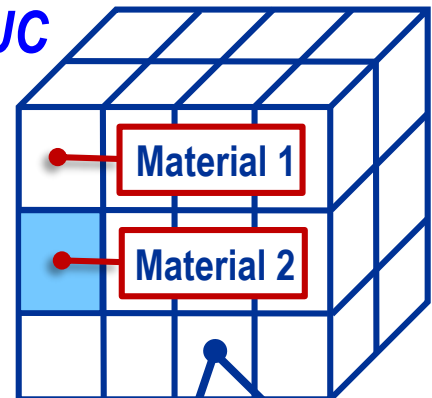
HFGMC (2000s)

- 2nd order displacement field in subcells
- Elastic stresses and strains piecewise linear
- Number of linear algebraic equations is rather large
- Local inelasticity/damage
- *Has shear coupling*
- Has “subcell mesh” sensitivity

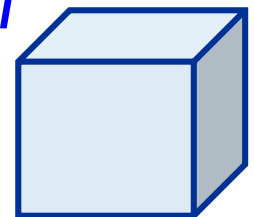
Repeating Unit Cell (RUC)
of composite material

- ❖ RUC made subcells
- ❖ Multiscale capability

RUC



Subcell



We currently only use GMC in FEAMAC/CARES

Aboudi, J.; Arnold, S.M.; and Bednarczyk, B.A. (2013) *Micromechanics of Composite Materials: A Generalized Multiscale Analysis Approach*, Elsevier, Oxford, UK.

Aboudi, J.; Pindera, M.J.; and Arnold, S.M. (2003): Higher-Order Theory for Periodic Multiphase Materials With Inelastic Phases. *Int. J. Plast.*, vol. 19, pp. 805–847.

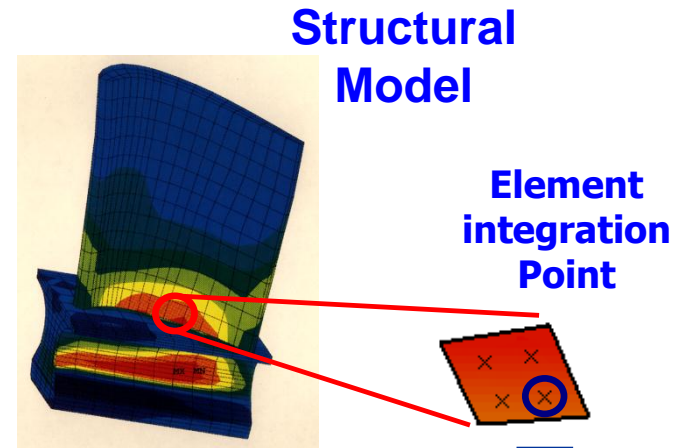
CARES: Ceramics Analyses and Reliability Evaluation of Structures

Life Prediction & Component Design Code For Advanced Ceramics

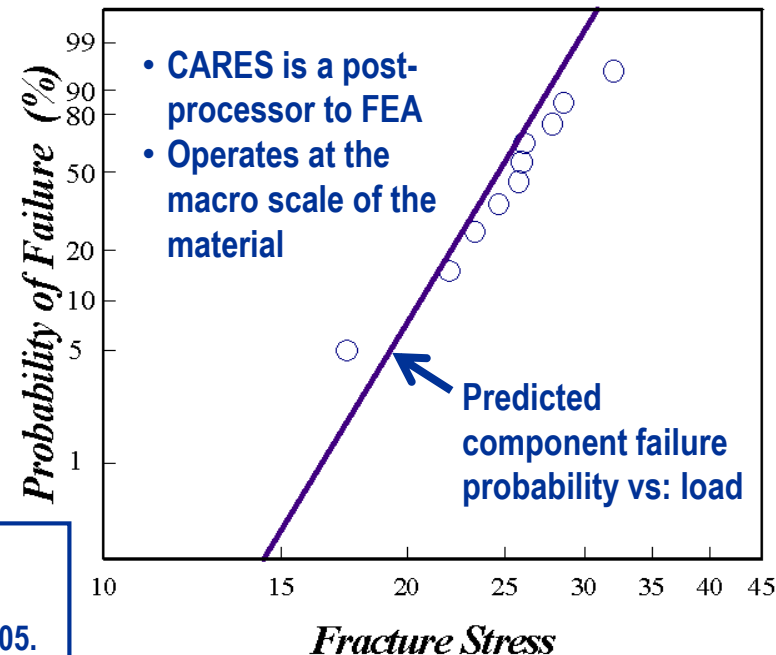
- Developed to predict the probability of failure of ceramic components under complex thermomechanical loading
- Combines Weibull & Weakest Link theory with concepts of linear elastic fracture mechanics (the *Unit Sphere* model)

Component Reliability Analysis Capability:

- Transient loads and temperatures
- Fast-Fracture Rupture
- Time-dependent (da/dt) crack growth
- Cycle-dependent (da/dn) crack growth
- Multiaxial stress failure models
(*PIA* & *Unit Sphere* & *Tsai-Wu* & *Tsai-Hill*)
- Proof test



(CARES)
reliability analysis



Nemeth, Jadaan, Gyekenyesi.: "Lifetime Reliability Prediction of Ceramic Structures Under Transient Thermomechanical Loads." NASA/TP-2005-212505, 2005.

Approach for Life Prediction & Component Design of Composites

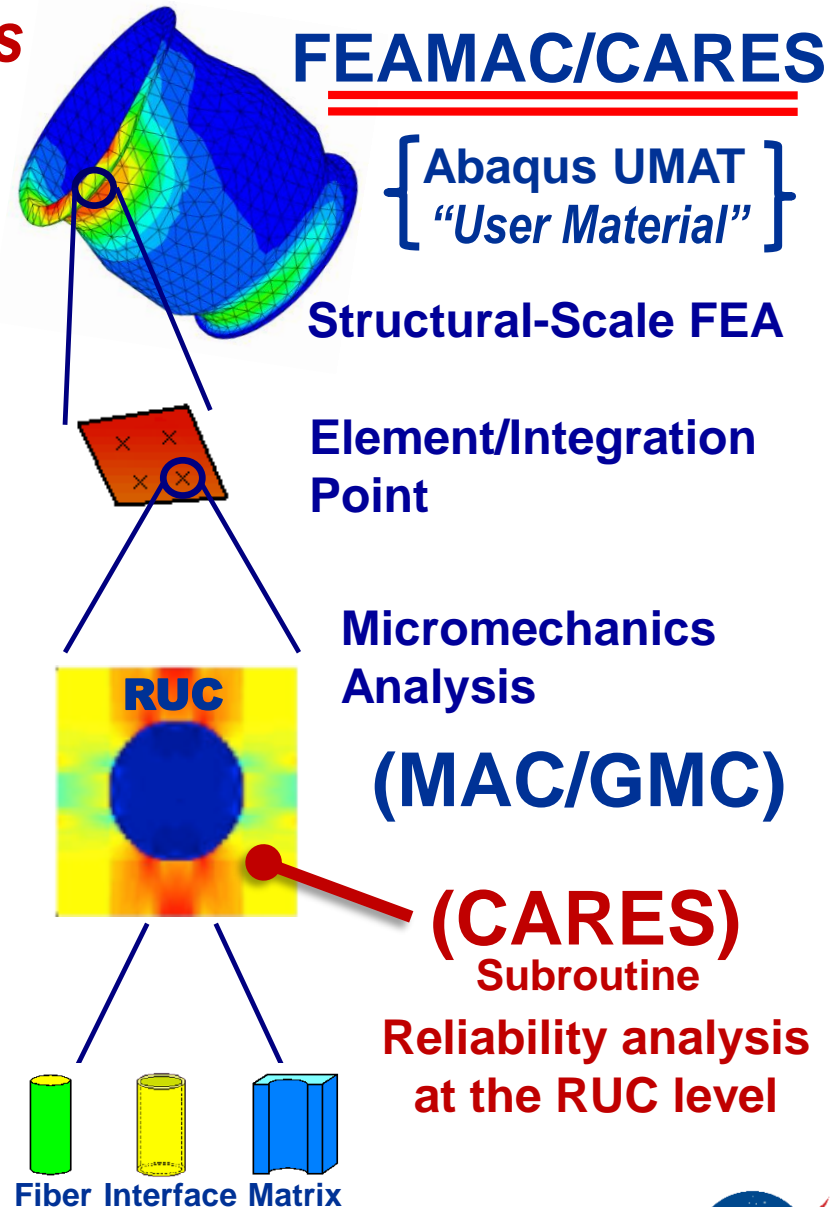
➤ *Combine CARES, MAC & FEA codes*

Move CARES from the macroscopic scale of the structure to the microscale of the individual RUC material constituents

❖ FEAMAC/CARES Capability:

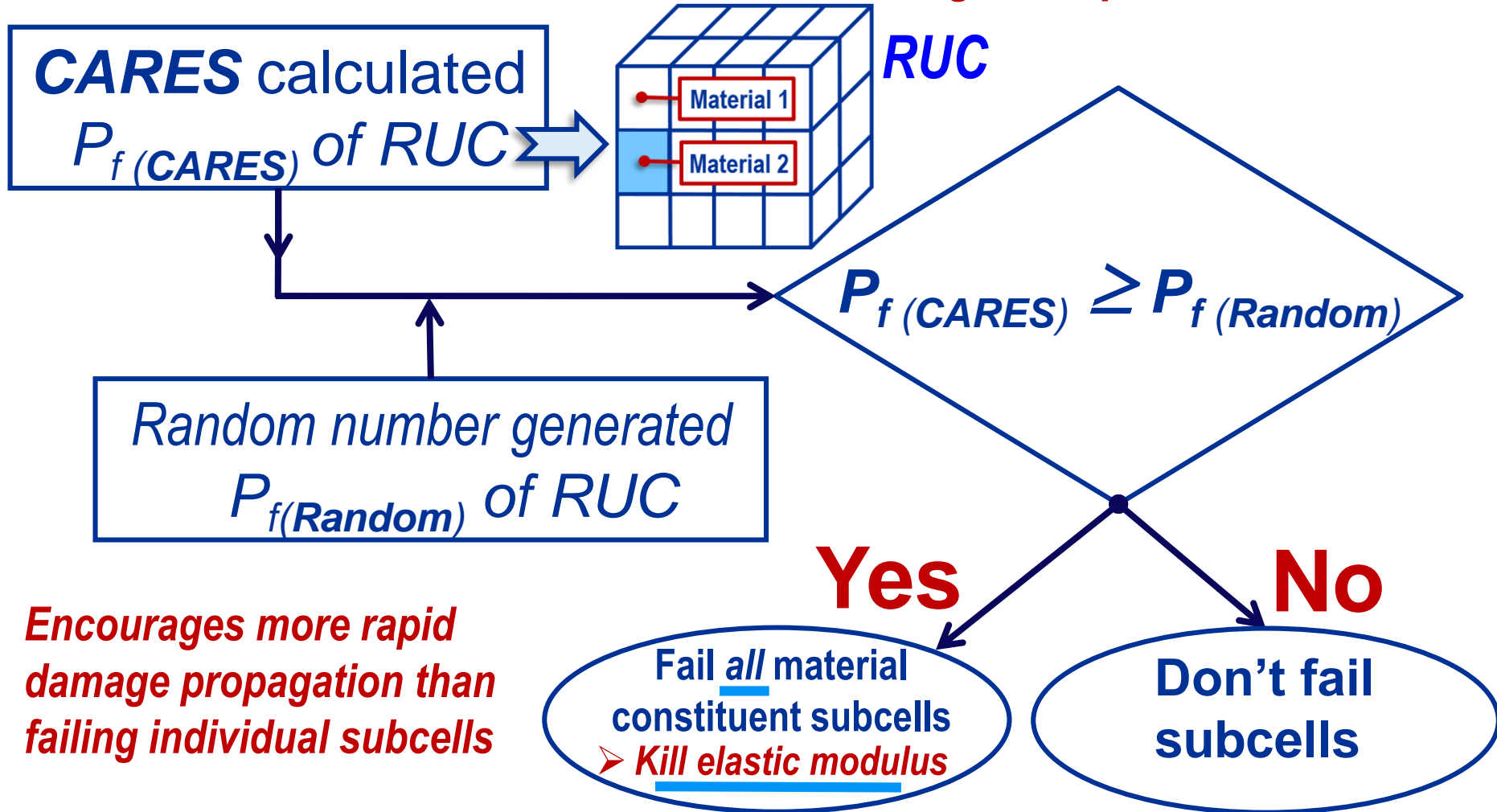
- Individual constituent and component level probability of failure tracked (for failure initiation)
- Individual & concurrent failure modes
- Laminate level analysis capability
- Progressive damage capability/simulation
- Subcells killed at random failure thresholds

Debonding/crack path physics at constituent level not explicitly included



Progressive Damage Criterion

Calculate failure probability, P_f , for each material constituent of the RUC associated with an element integration point



Encourages more rapid damage propagation than failing individual subcells

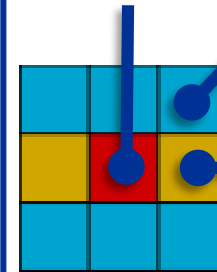
Random Element Failure vs: Neighbor Influenced Failure (Cellular Automaton Enhancement)

Encourage more abrupt failure and “crack-like” damage growth patterns

A cellular automaton is a collection of “colored” cells on a grid that evolves through discrete time steps according to a set of rules based on the states of neighboring cells

Rule: When failure of an element is encountered, the random failure threshold of the neighboring elements are adjusted to that of the failed element. Load state determines which elements have highest probability of failure

Failed element



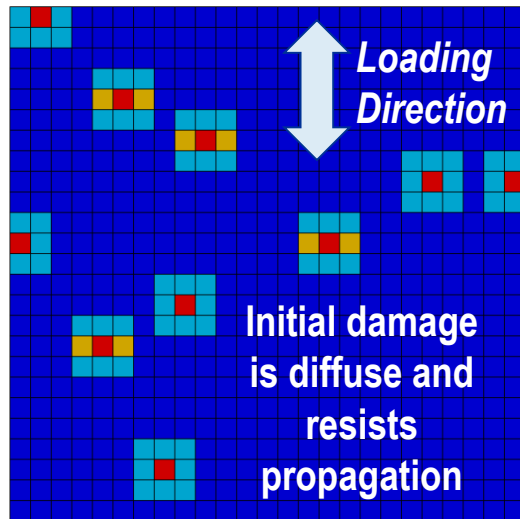
Adjacent element

Adjacent 2 elements with highest

P_f (CARES) has

P_f (Random) adjusted

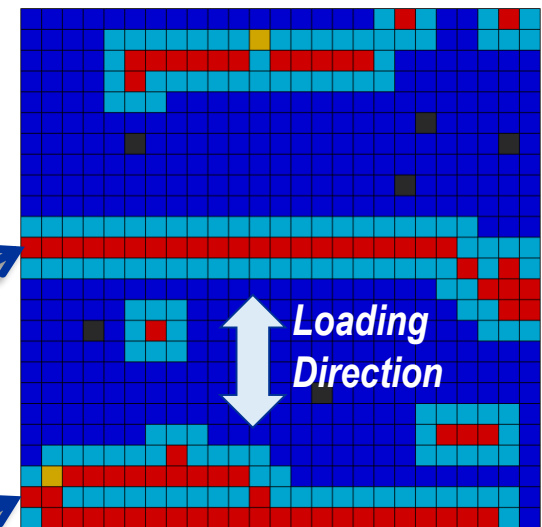
Random element failure



**Example: 0° Ply
uniaxial ramp load
25x25 FEA mesh**

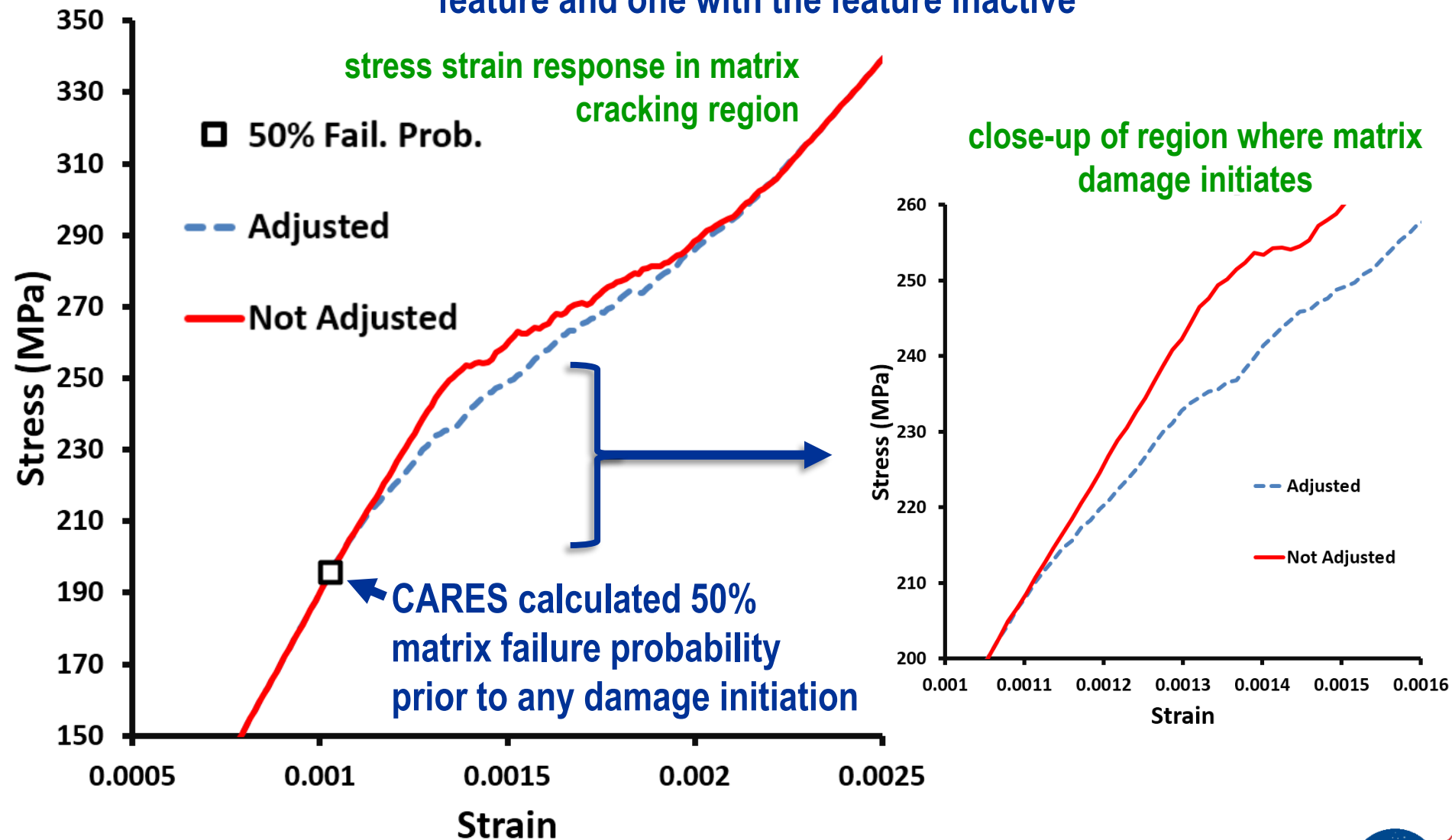
Adjusted element P_f (Random) more likely to be lower than original P_f (Random) and fail sooner as load increases – *enhancing damage propagation*

Cellular automaton



0° single ply tensile specimen *(Load parallel to fiber axis)*

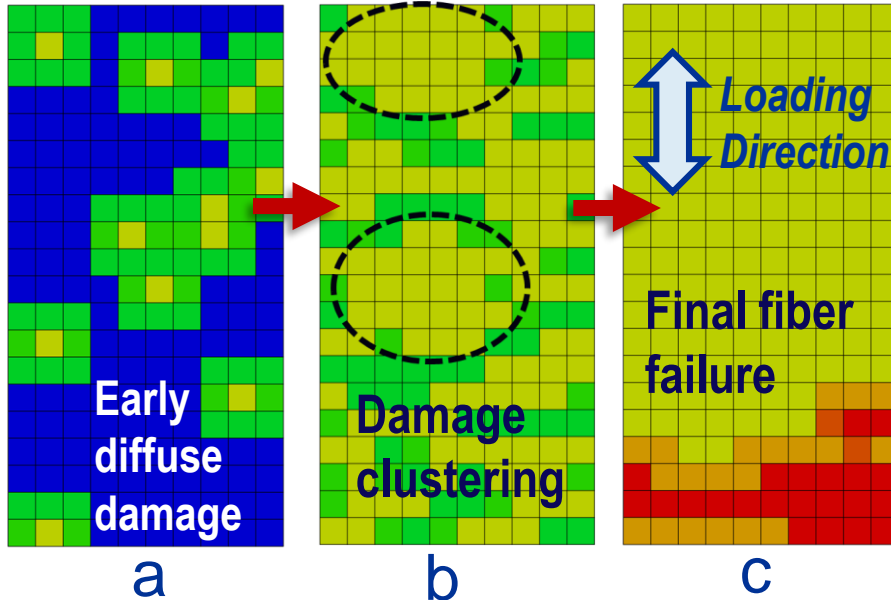
Shown are two trial executions; one using the automaton adjusted element feature and one with the feature inactive



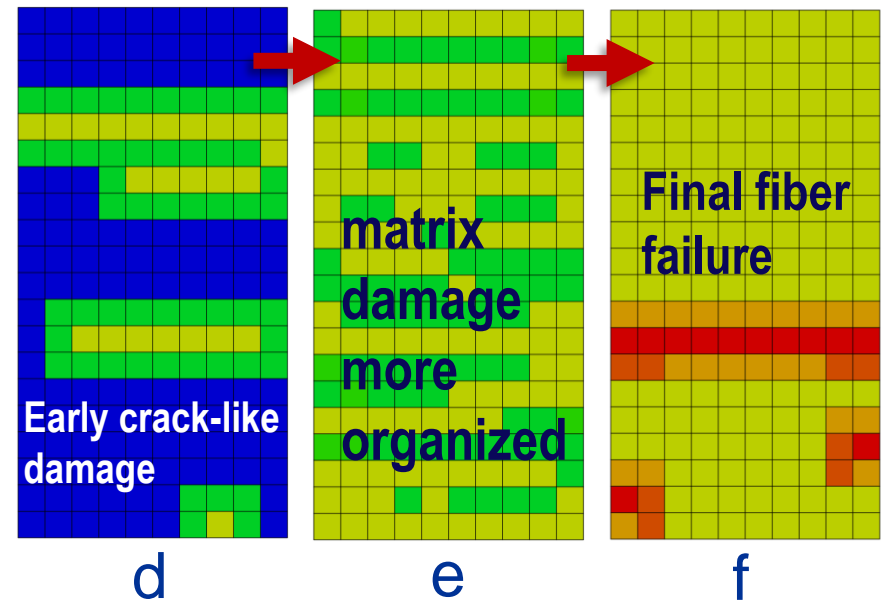
0° single ply tensile specimen

Progression of damage in FE model of a unidirectional ply under longitudinal loading

Not Adjusted



Automaton Adjusted



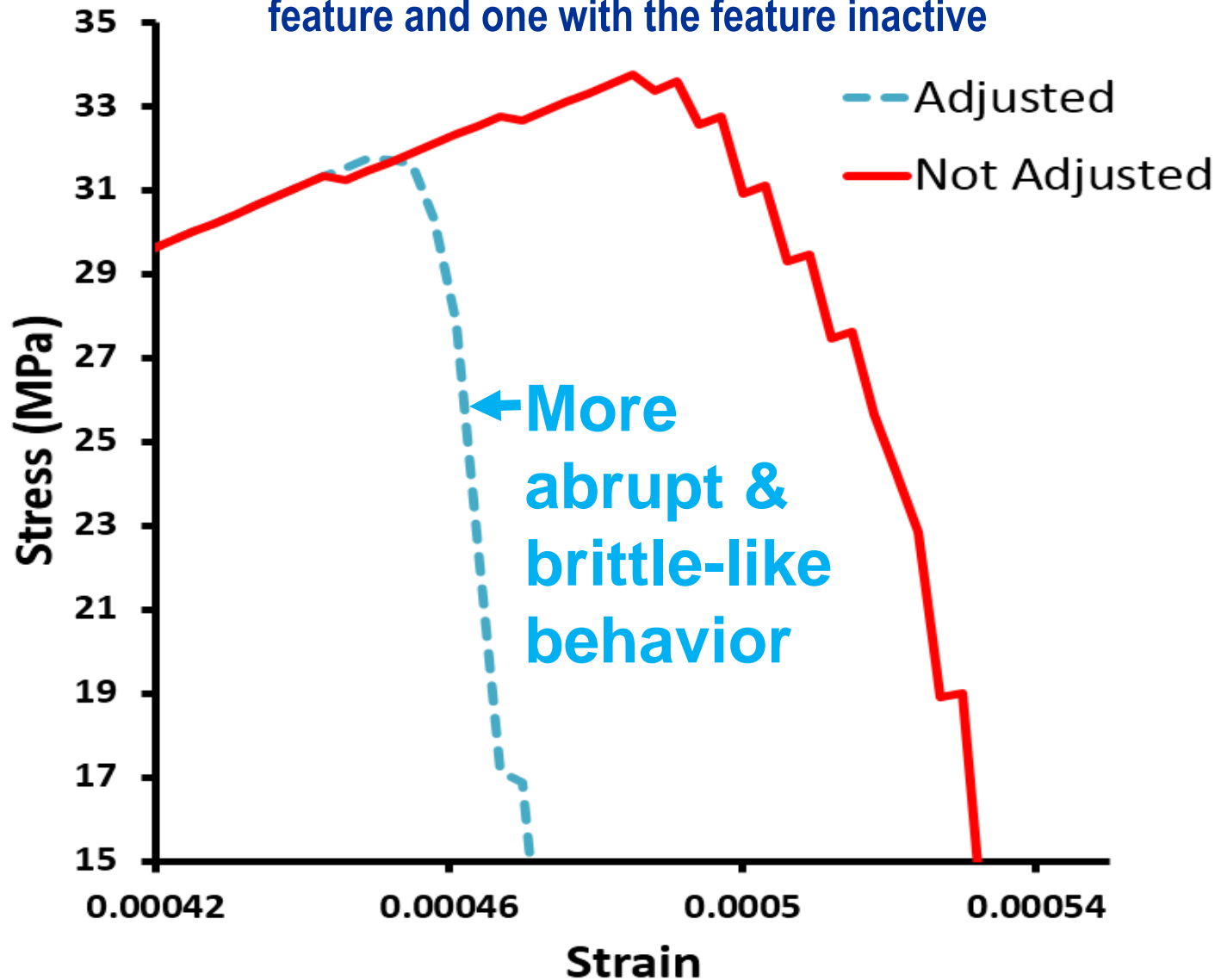
(a) and (d) ; early matrix damage

(b) and (e) ; progression to substantial matrix damage

(c) and (f) ; final composite failure (fiber failure)

90° single ply tensile specimen (*Load **transverse** to fiber axis*)

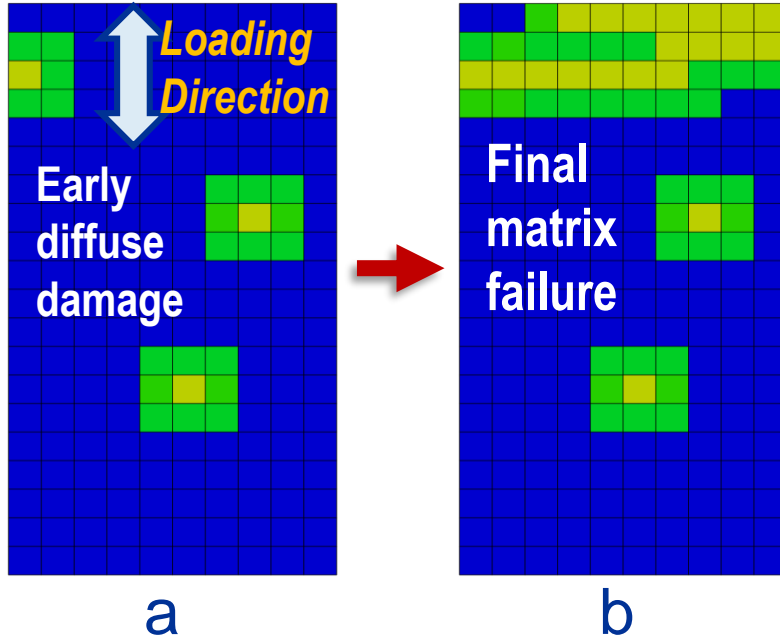
Shown are two trial executions; one using the automaton adjusted element feature and one with the feature inactive



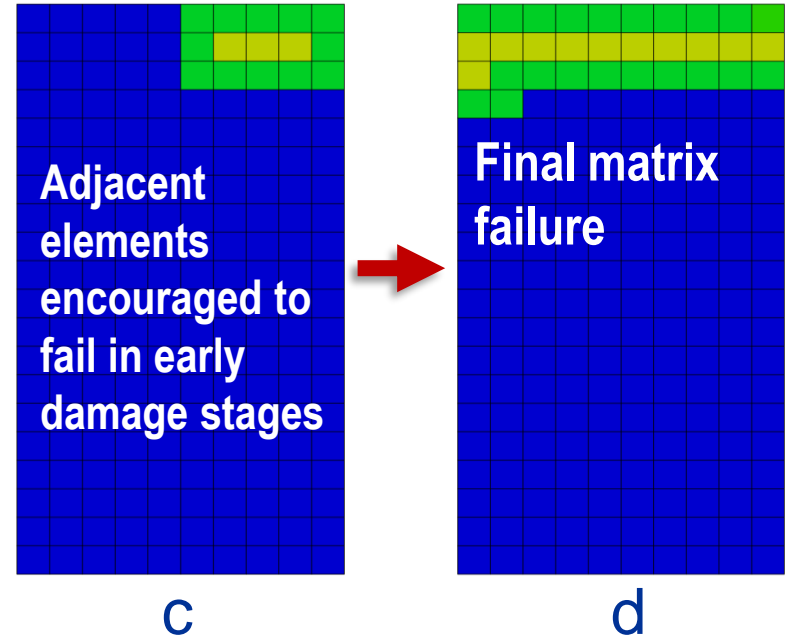
90° single ply tensile specimen

Progression of damage in FE model of a unidirectional ply under *transverse* loading

Not Adjusted



Automaton Adjusted



(a) and (c) ; early matrix damage
(b) and (d) ; final composite failure (matrix failure)

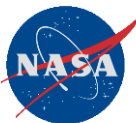
Example: SiC/RBSN Laminated Composite in *On-Axis* & *Off-Axis* Loading

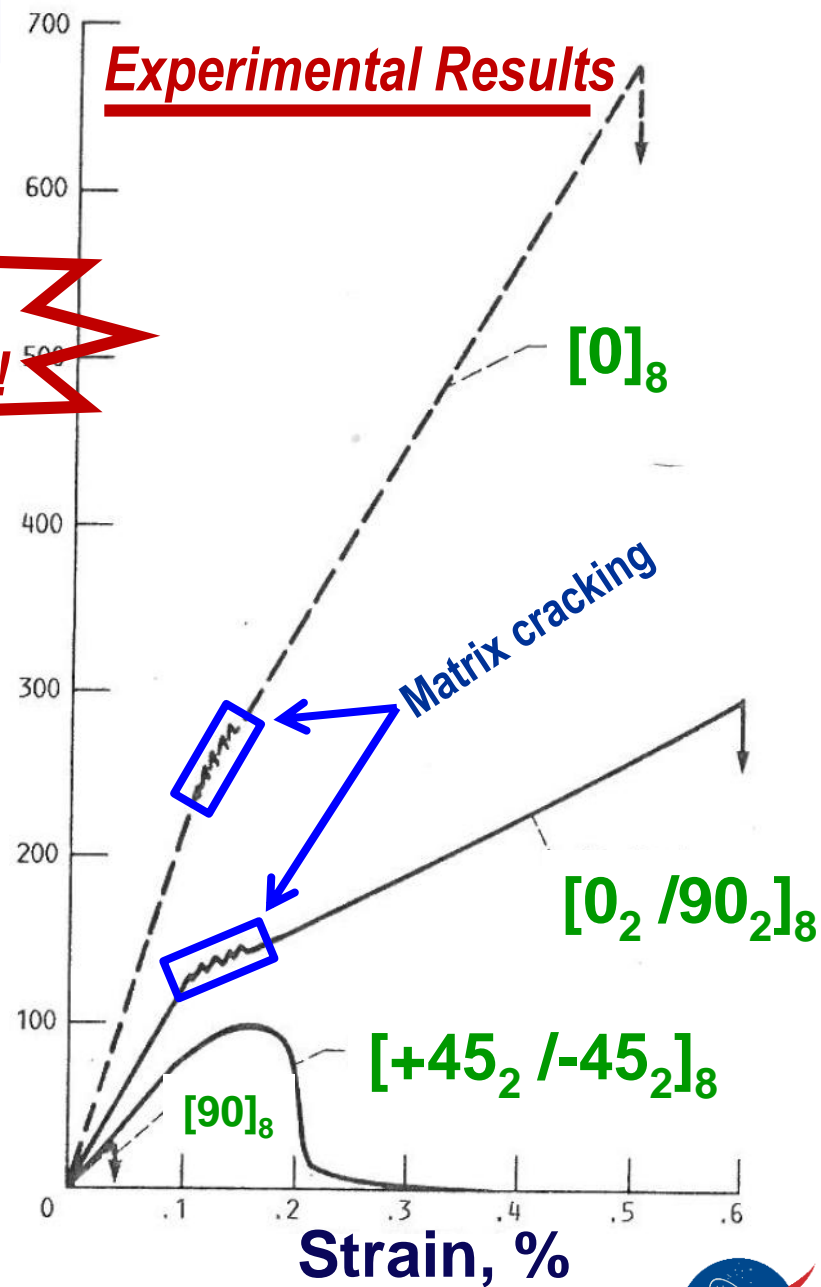
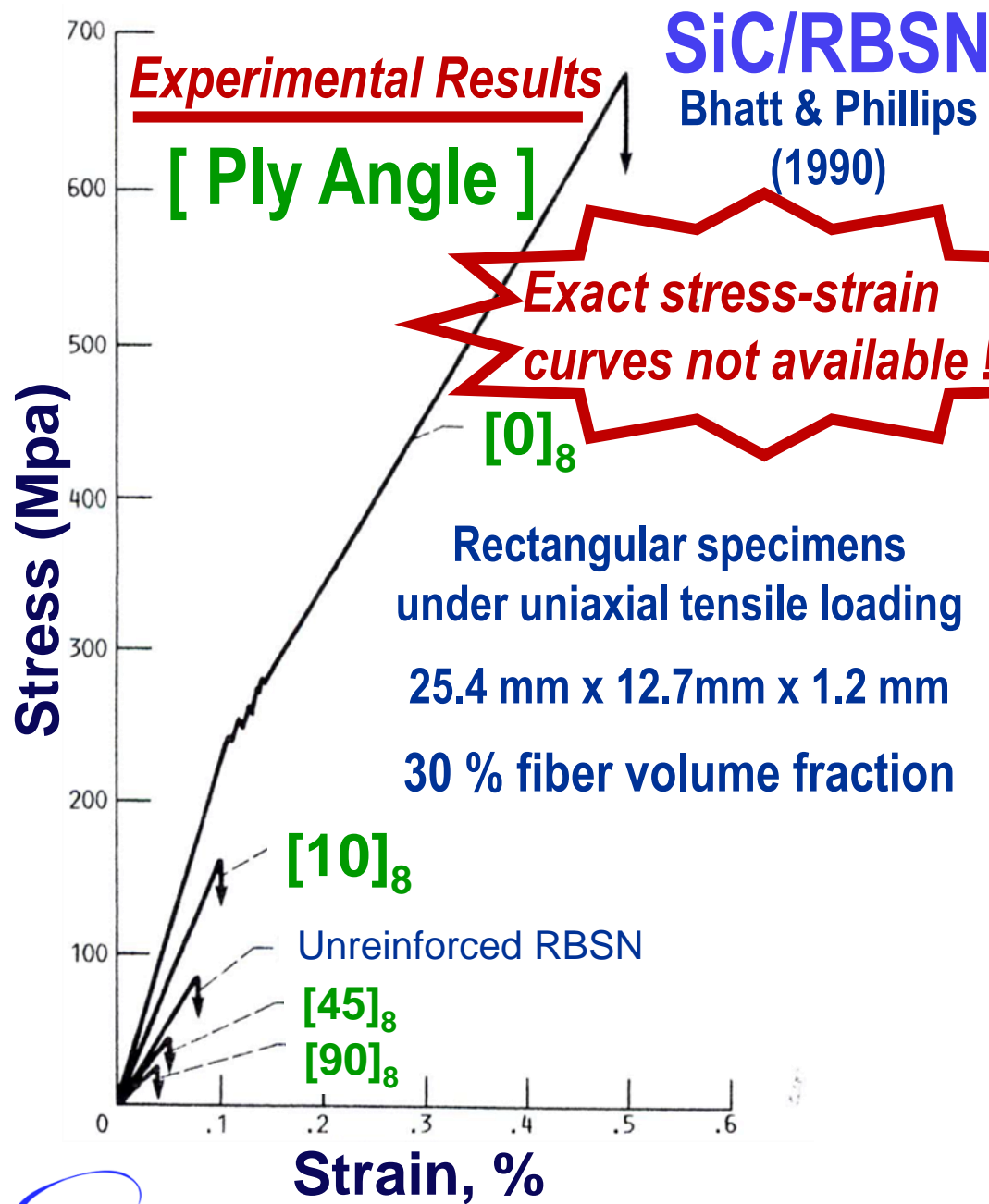
- Tested by Bhatt & Phillips (1990)

➤ displays key mechanisms/features for model material

- SCS-6 fiber/Reaction Bonded Silicon Nitride matrix composite examined in detail by NASA ➤ several papers published
- Laminated CMCs of interest to industry and less complex than woven composites
 - failure modes are not conflicted with complex fiber architecture
- [0] & [0/90] laminates display nonlinearity due to matrix failure, followed by fiber failure.
- Remaining ply orientations display sudden brittle failure.

Bhatt, R.T., and Phillips, R.E.: "Laminate Behavior for SiC Fiber-Reinforced Reaction-Bonded Silicon Nitride Matrix Composites." J. of Comp. Tech. & Res. V. 12, No. 1, Spring 1990, pp. 13-23.





SiC/RBSN Example Procedure & Setup

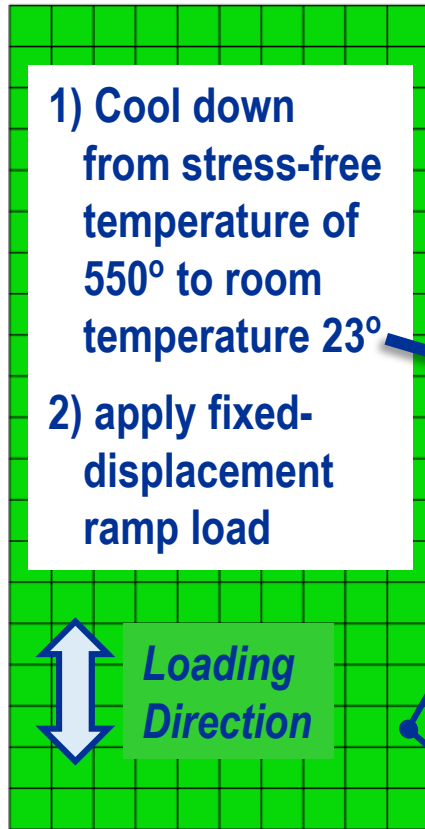
Abaqus FEA

S4 Shell elements

Fixed-displacement ramp load

Stochastic strength analysis:

(from individual trials / simulations / realizations)

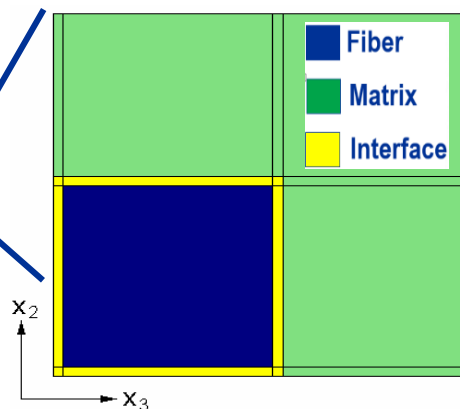


(10x20 mesh)

- ❖ Use **CARES Unit Sphere** failure criterion
 - assume **Isotropic material constituent strength**
– for simplicity and initial testing

*Residual stresses
in constituents*

MAC/GMC RUC



- Weibull parameters correlated to experimental results for 0° tensile specimen
- Interface strength made large:
 - Encourage matrix to fail before interface

▪ **Interfacial failure modes and sliding resistance not considered**

Constituent properties of SiC/RBSN with anisotropic thermal expansion coefficients

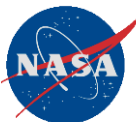
Constituent	Modulus, GPa	Poisson ratio	Longitudinal coefficient of thermal expansion, α_L (m/m/°C)	Transverse coefficient of thermal expansion, α_T (m/m/°C)
Fiber	390	0.17	4.1×10^{-6}	1.84×10^{-6}
Matrix	110	0.22	2.2×10^{-6}	2.2×10^{-6}
Interface	1.8	0.22	2.0×10^{-6}	2.0×10^{-6}

Assumed Weibull Parameters:

Fiber $m_V = 20$ $\sigma_{oV} = 2875 \text{ Mpa} \bullet m^{3/20}$

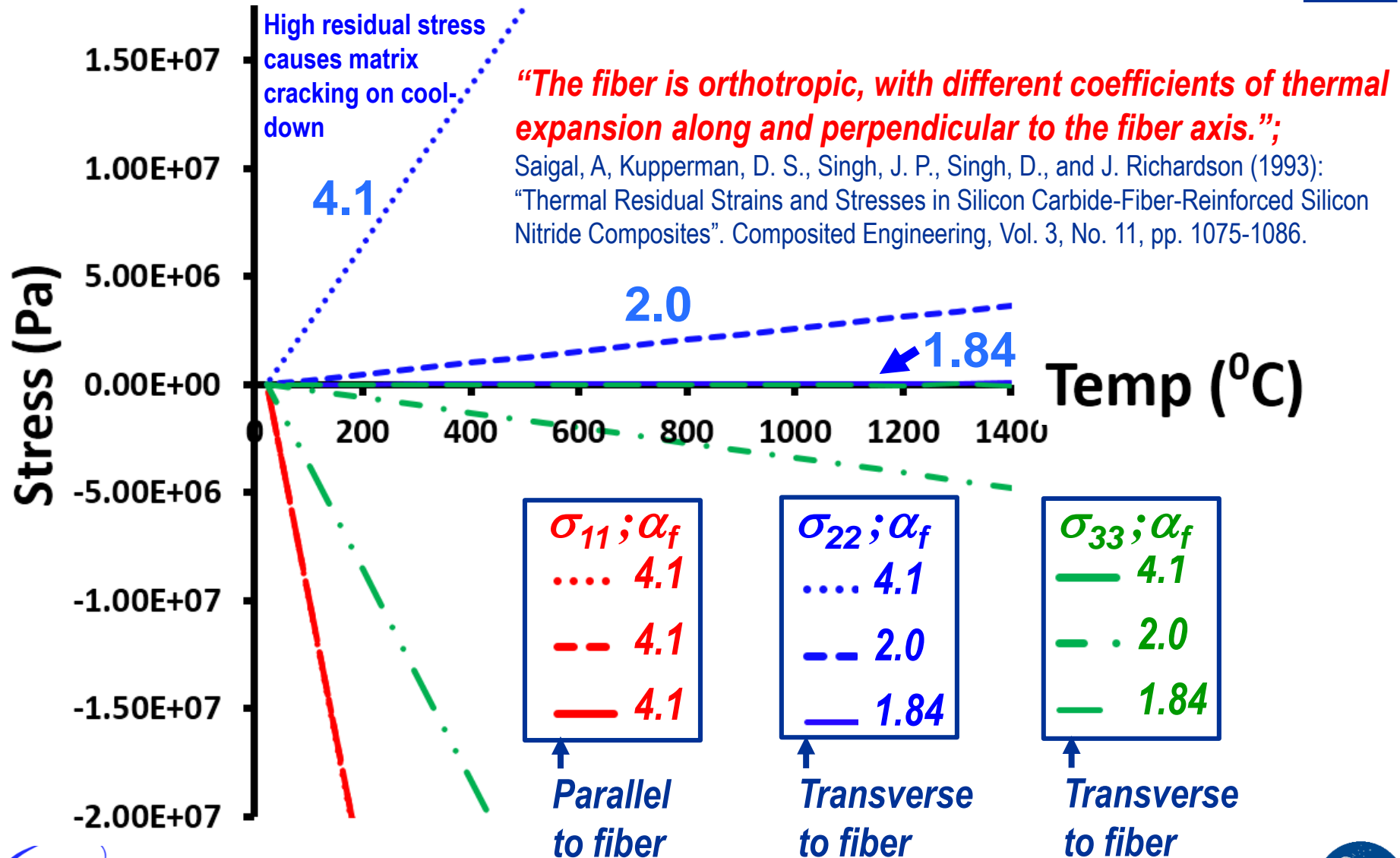
Matrix $m_V = 5.0$ $\sigma_{oV} = 150 \text{ Mpa} \bullet m^{3/5}$

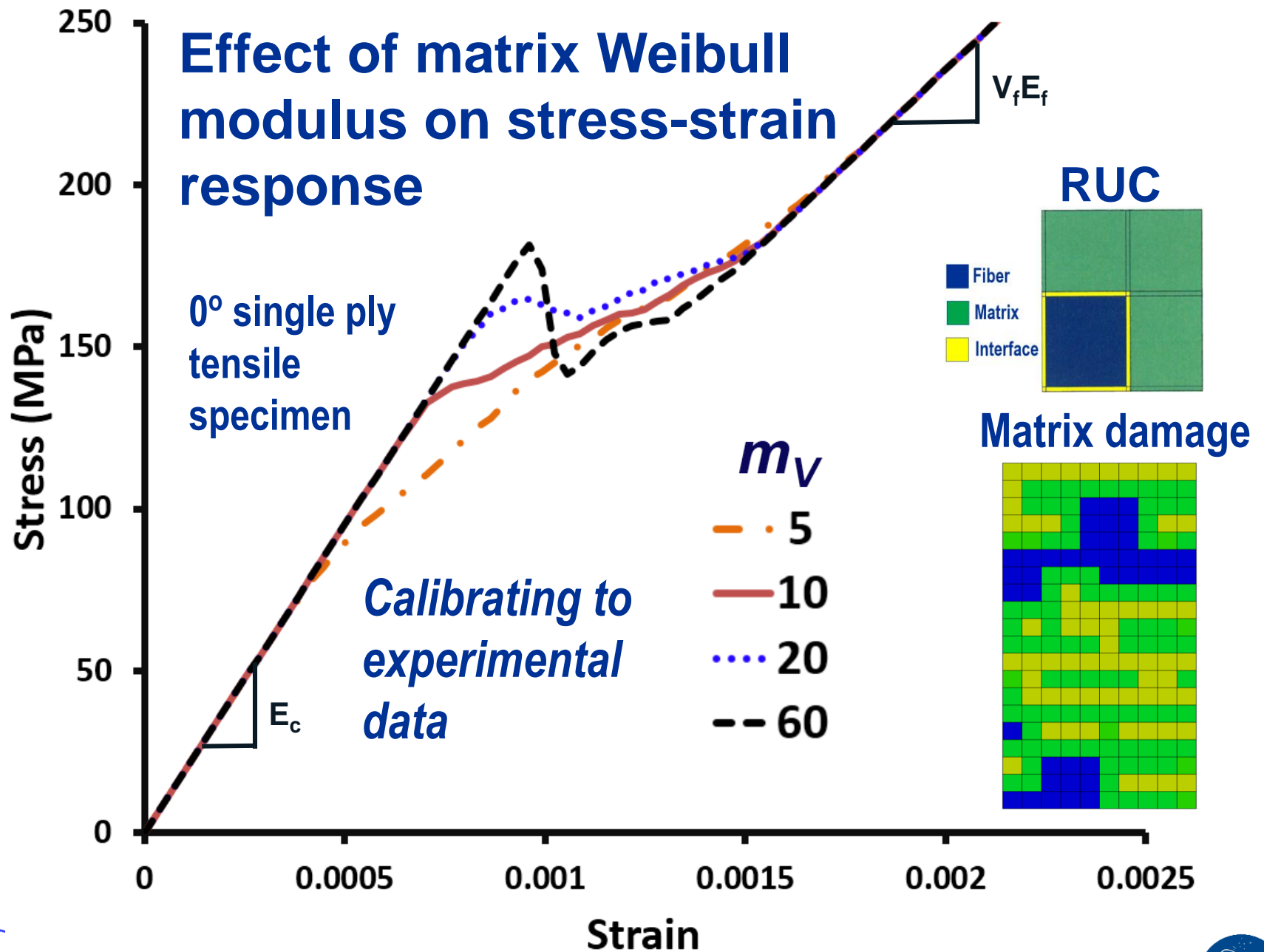
Interface $m_V = 5.0$ $\sigma_{oV} = 80 \text{ Mpa} \bullet m^{3/5}$

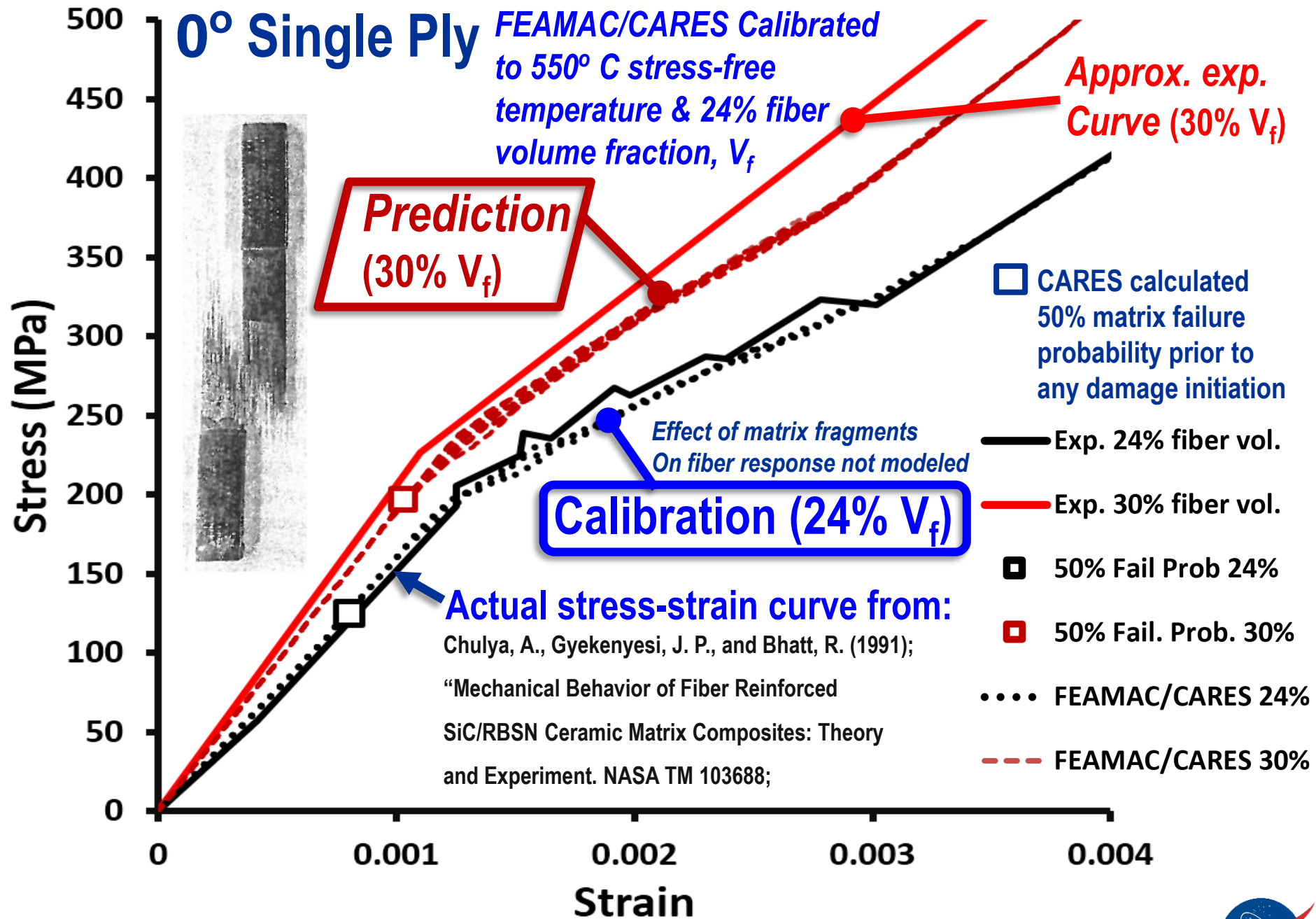


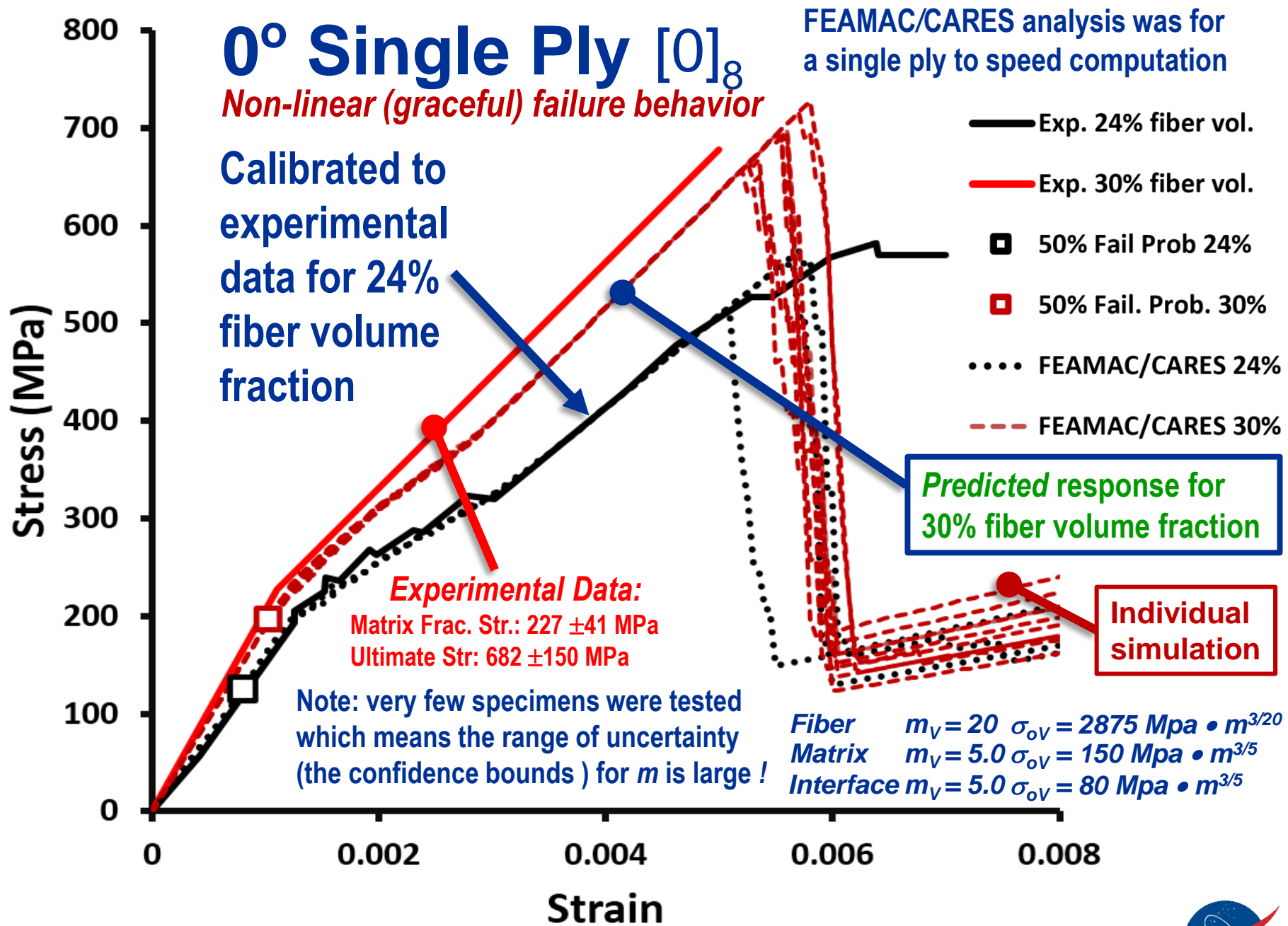
Residual matrix stresses after cool-down from temperature

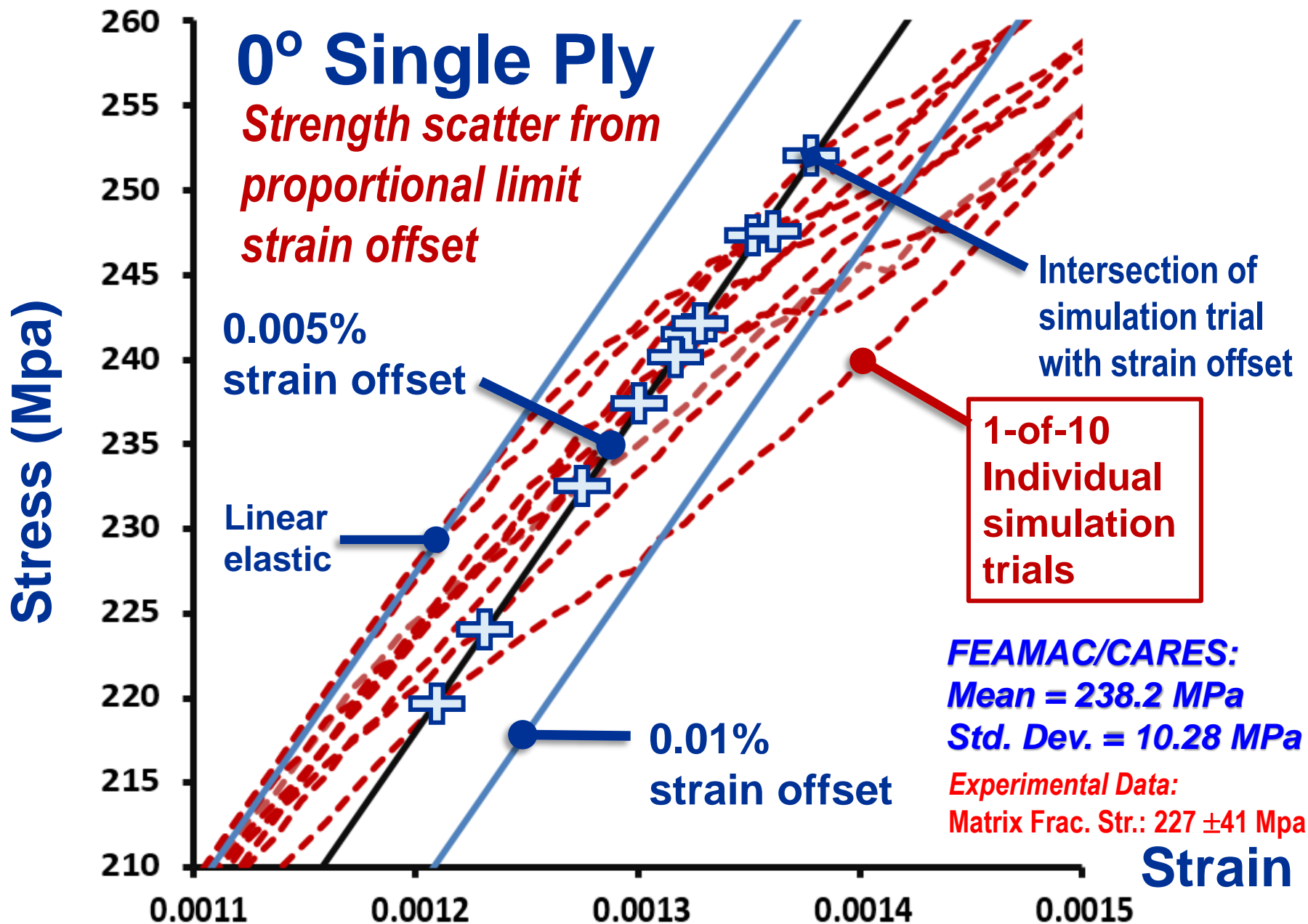
- Effect of anisotropic fiber-thermal-expansion-coefficient, α_f on RUC







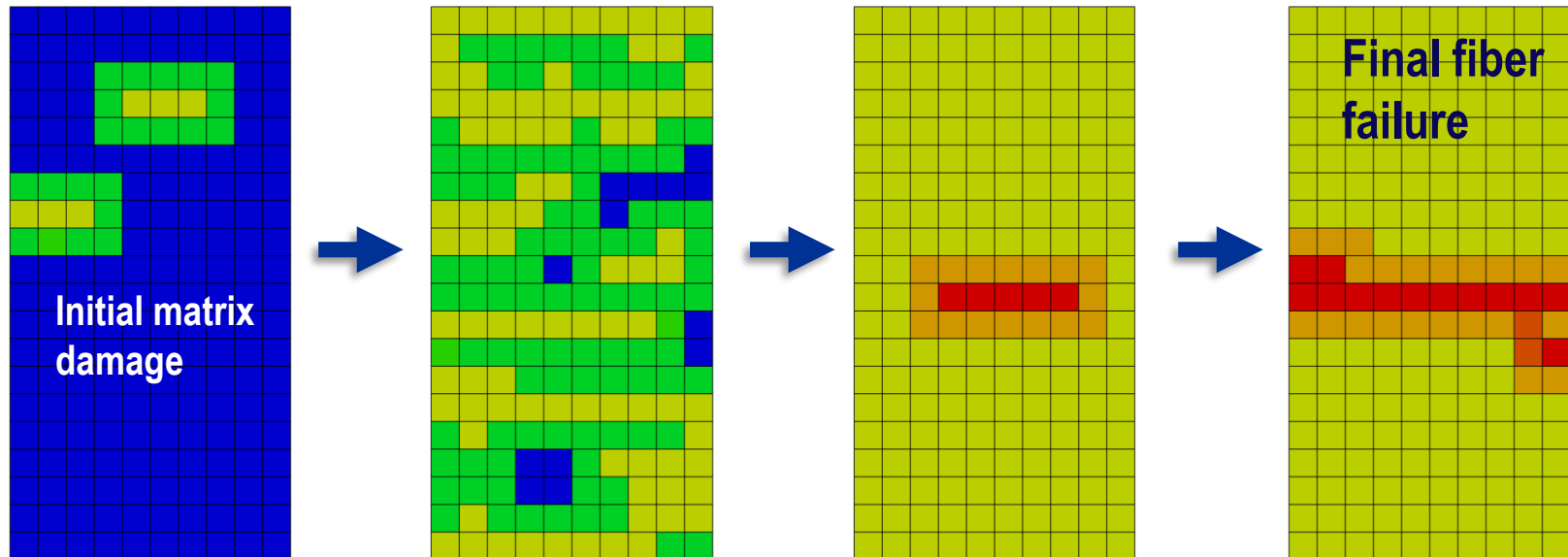
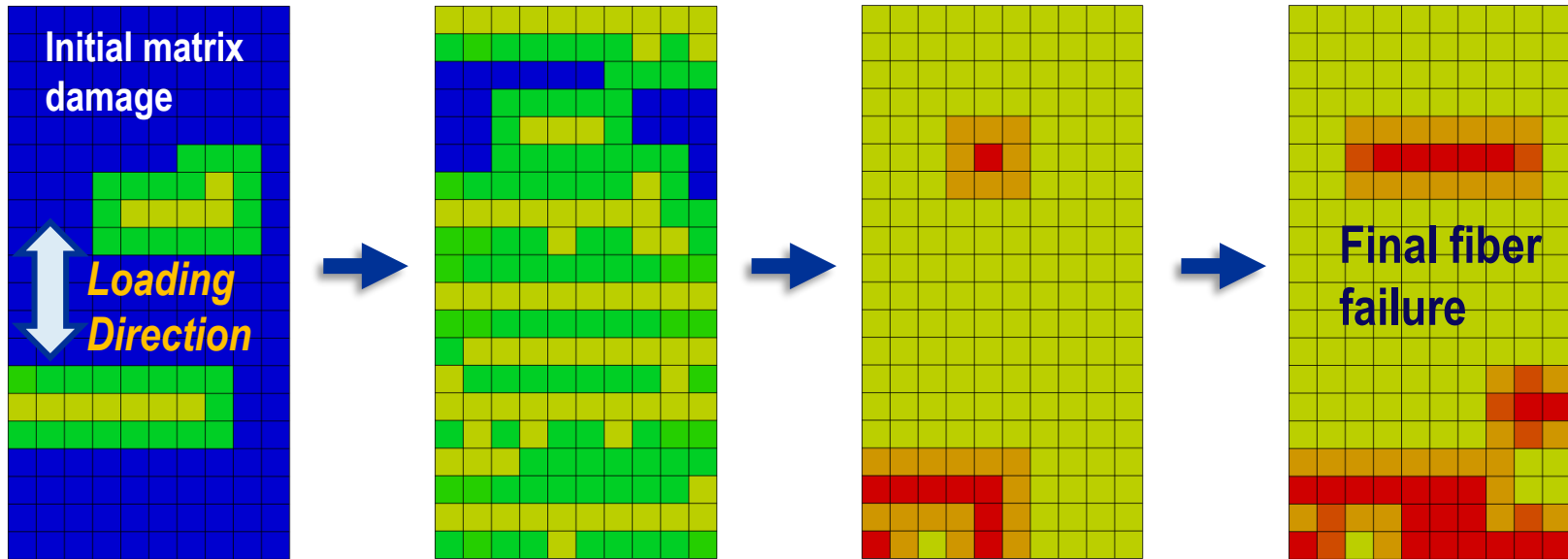




PLS is defined as the stress at 0.005% strain offset:

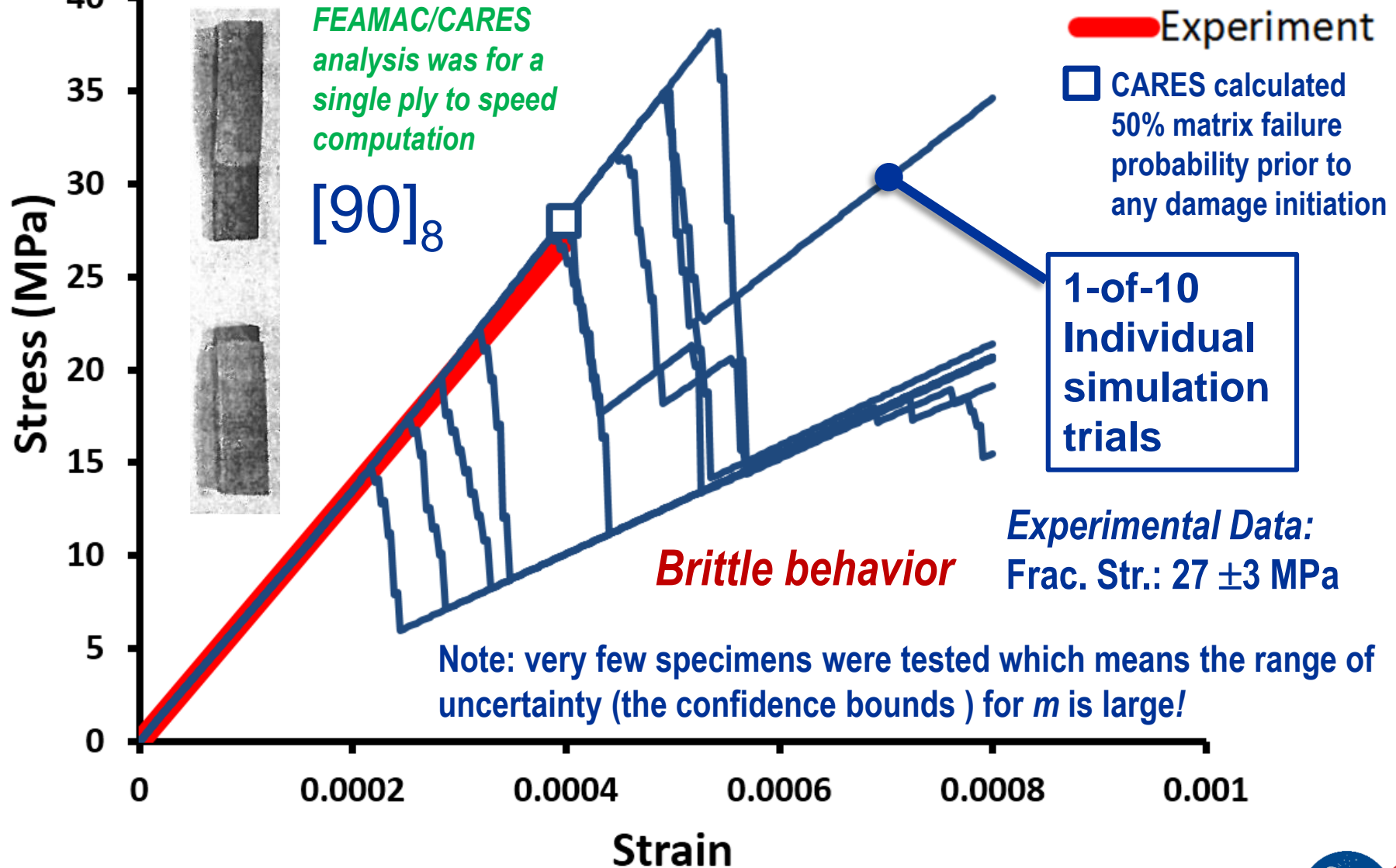
Kalluri, S; Calomino, A; and Brewer, D., "Computation of Variability in the Average Thermal and Mechanical Properties of a Melt-Infiltrated SiC/SiC Composite", High Temperature Ceramic Matrix Composites 5, M. Singh, R.J. Kearns, E. Lara-Curzio, R. Naslain, Eds, 2004, pp. 279-284

Damage progression of 0° tensile specimen - two trials (undeformed plot)

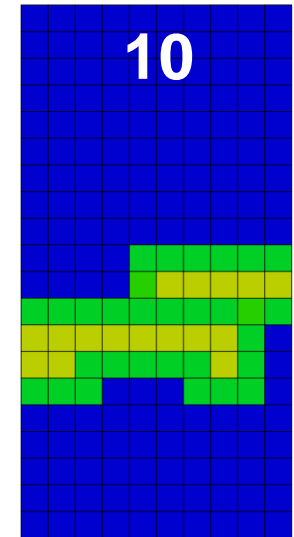
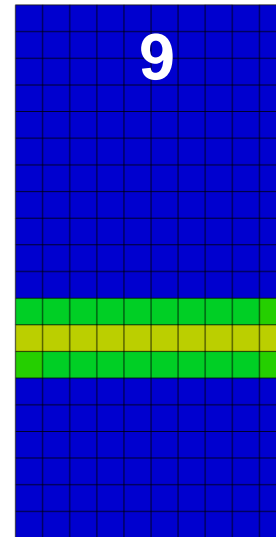
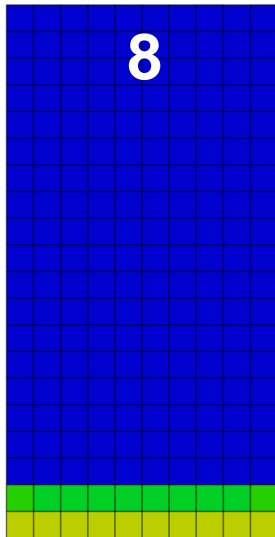
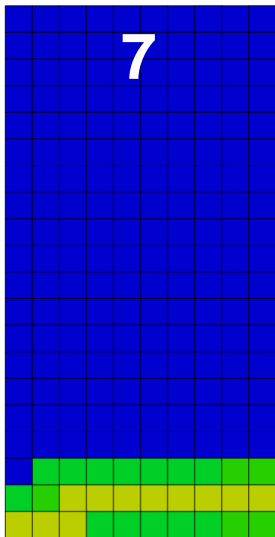
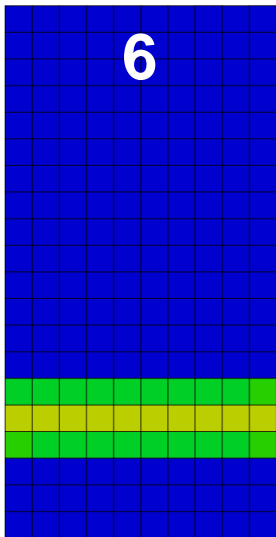
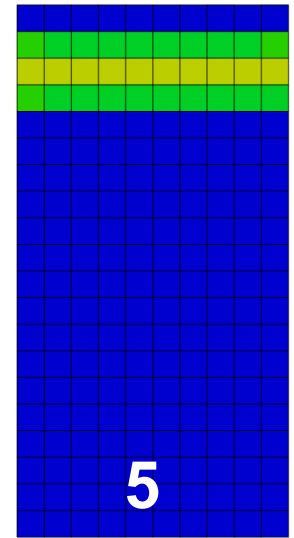
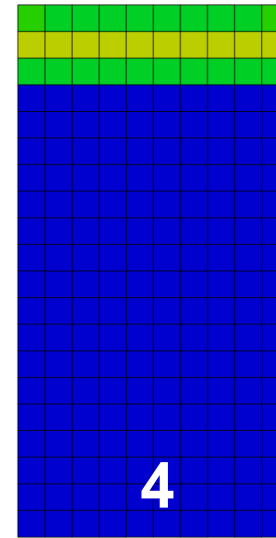
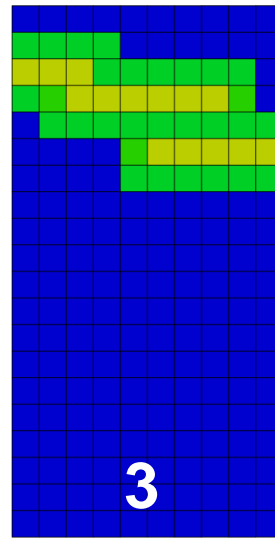
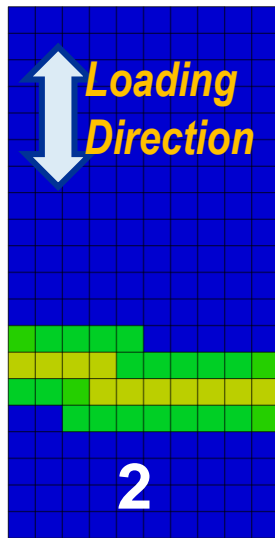


Prediction for Ten Trials for 90° Fiber Orientation

Assuming matrix and interface are isotropic strength materials

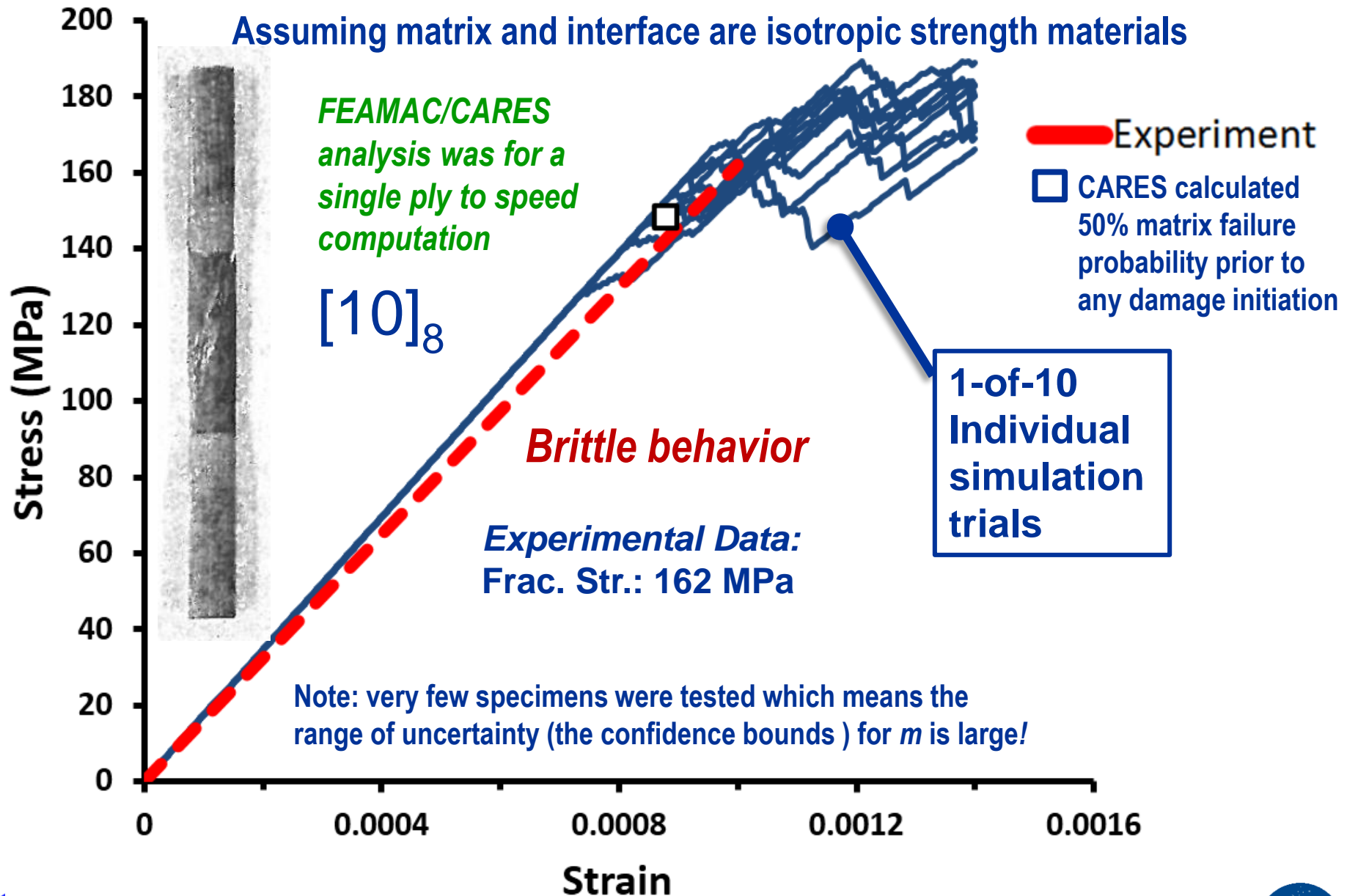


90° Tensile specimen *at final failure for 10 trials* – Undeformed plots



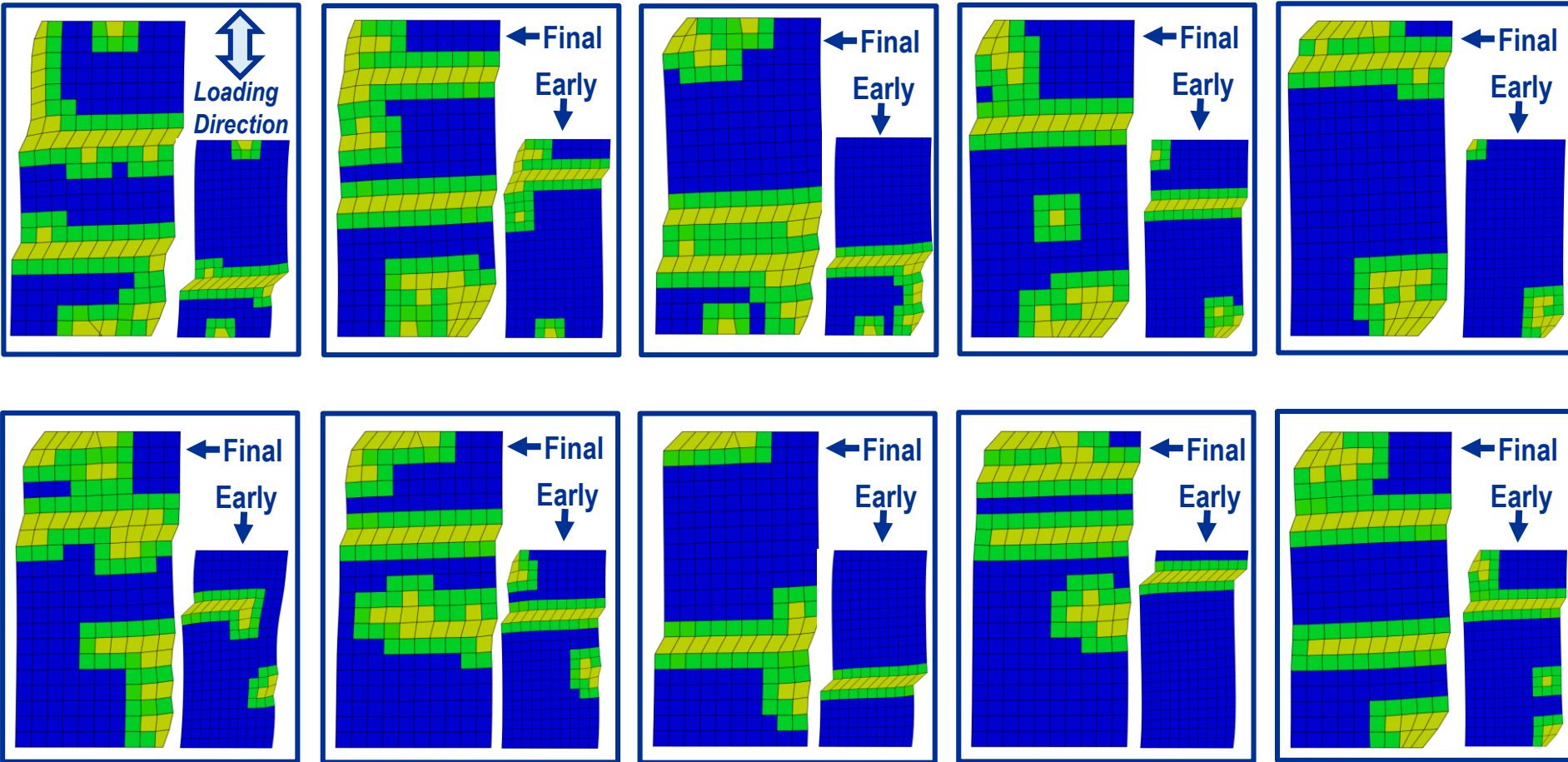
Prediction for Ten Trials for 10° Fiber Orientation

Assuming matrix and interface are isotropic strength materials



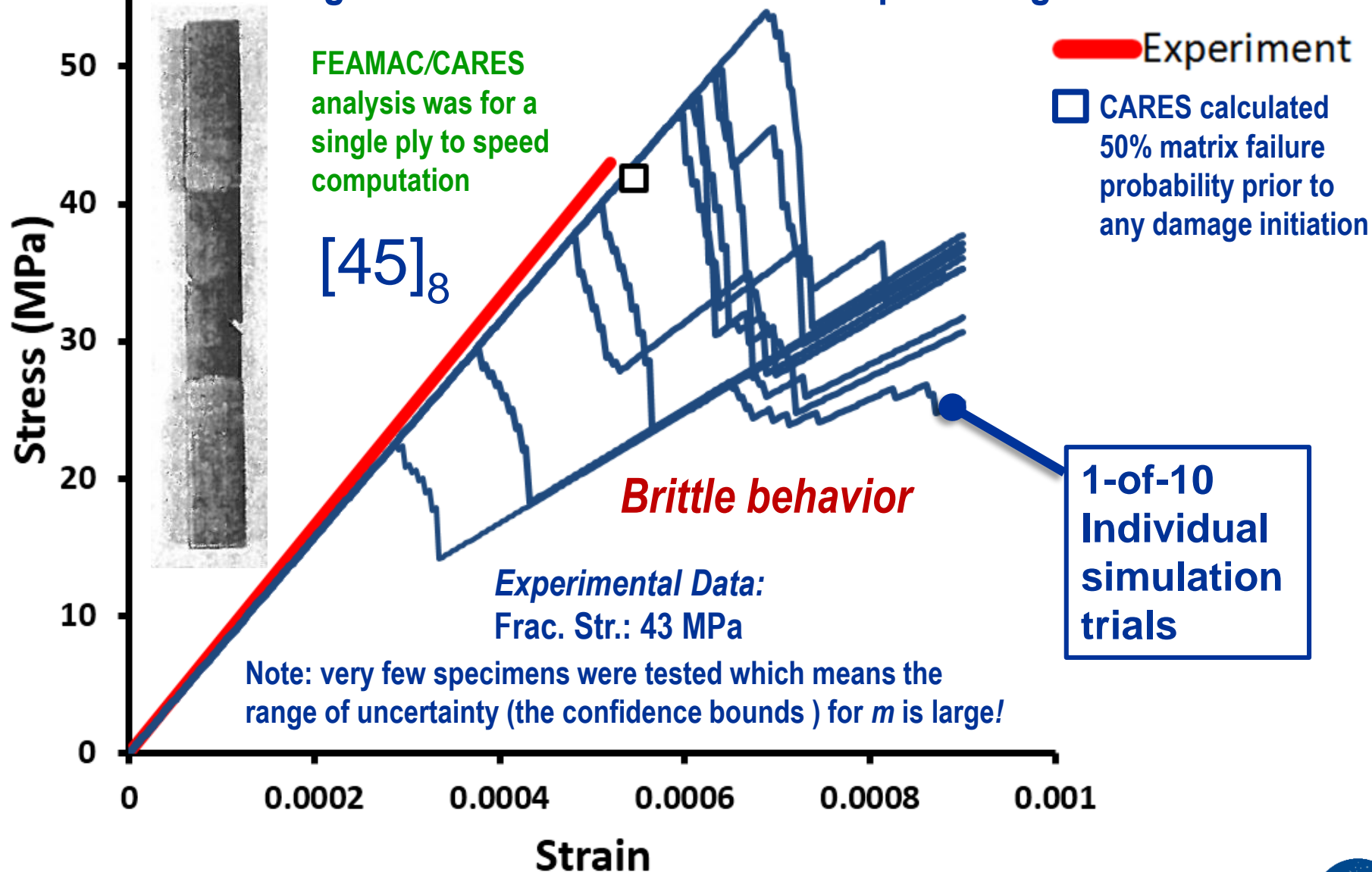
10⁰ off-axis tensile specimen; 10 trials at final (matrix) failure; deformed plots

- Edges are allowed to freely deform (warp) on cool-down
- *After cool-down*; bottom edge fixed in loading direction when displacement load applied
- *After cool-down*; single node along top edge (middle) fixed in direction perpendicular to displacement direct.



Prediction for Ten Trials for 45° Fiber Orientation

Assuming matrix and interface are isotropic strength materials

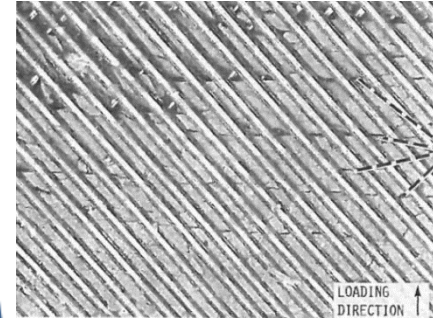


Prediction for Ten Trials for $[+45_2 / -45_2]_8$ Fiber Orientation

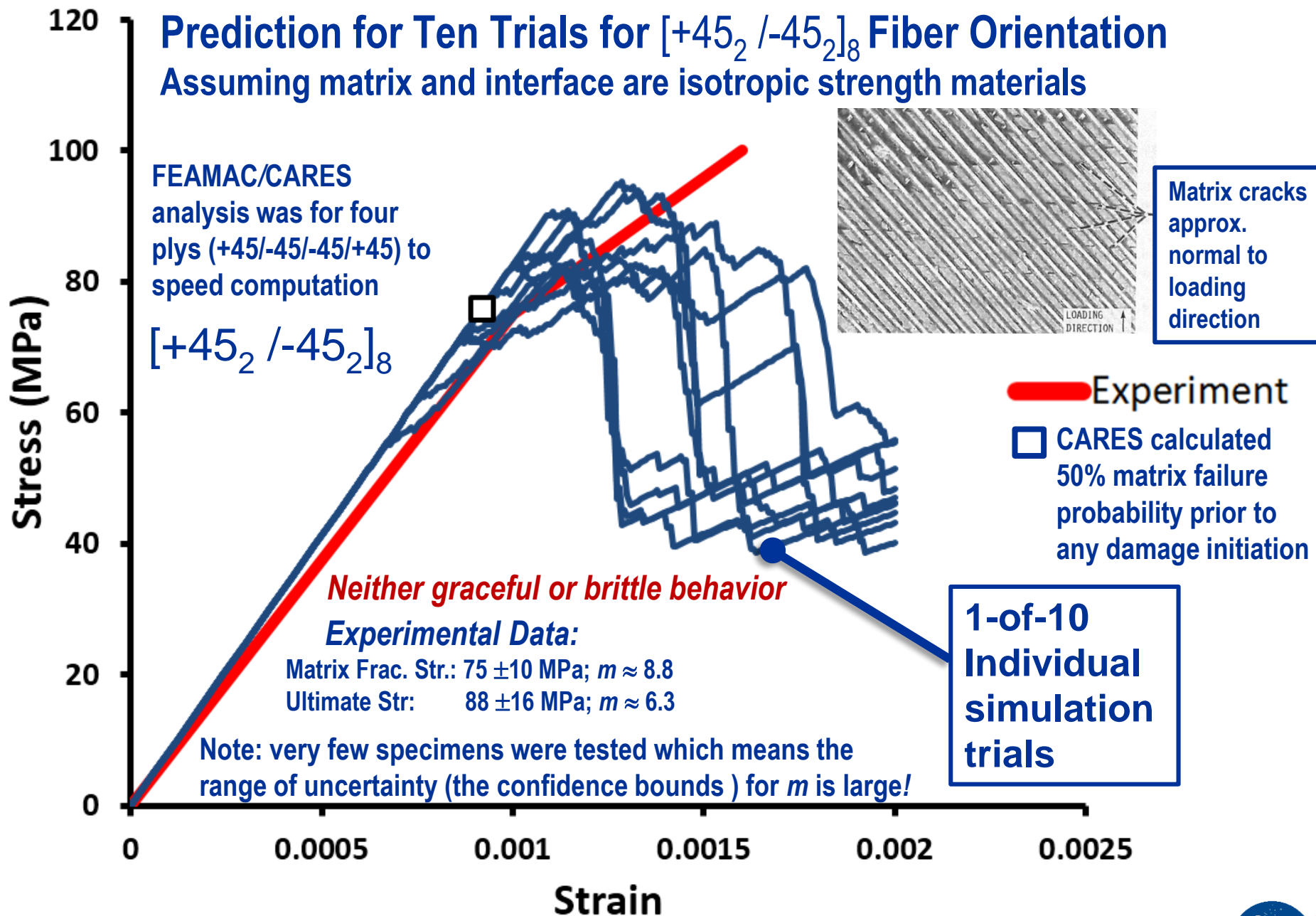
Assuming matrix and interface are isotropic strength materials

FEAMAC/CARES
analysis was for four
plys (+45/-45/-45/+45) to
speed computation

$[+45_2 / -45_2]_8$



Matrix cracks
approx.
normal to
loading
direction



Experiment
CARES calculated
50% matrix failure
probability prior to
any damage initiation

1-of-10
Individual
simulation
trials

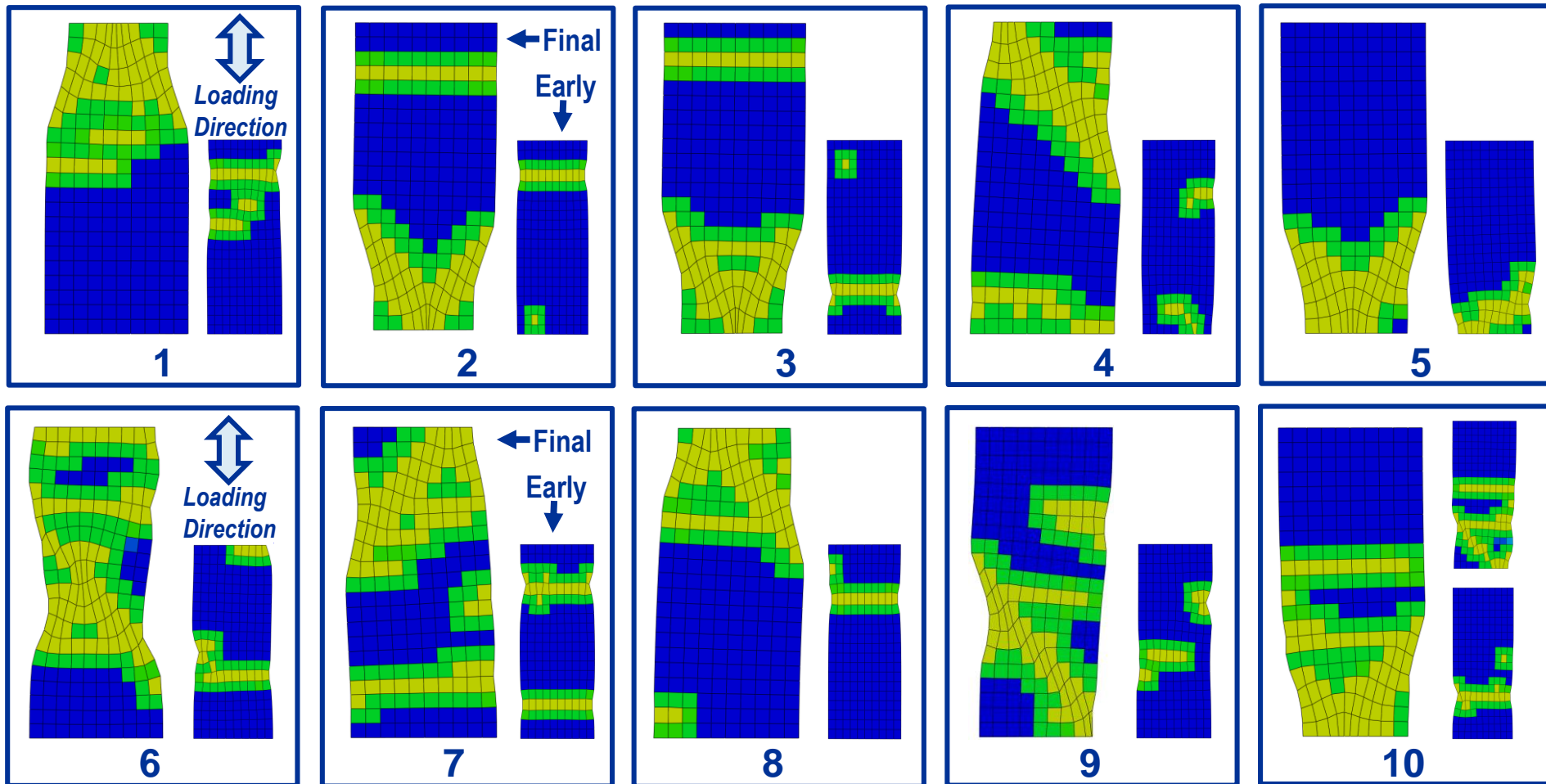
Neither graceful or brittle behavior
Experimental Data:

Matrix Frac. Str.: 75 ± 10 MPa; $m \approx 8.8$
Ultimate Str: 88 ± 16 MPa; $m \approx 6.3$

Note: very few specimens were tested which means the
range of uncertainty (the confidence bounds) for m is large!

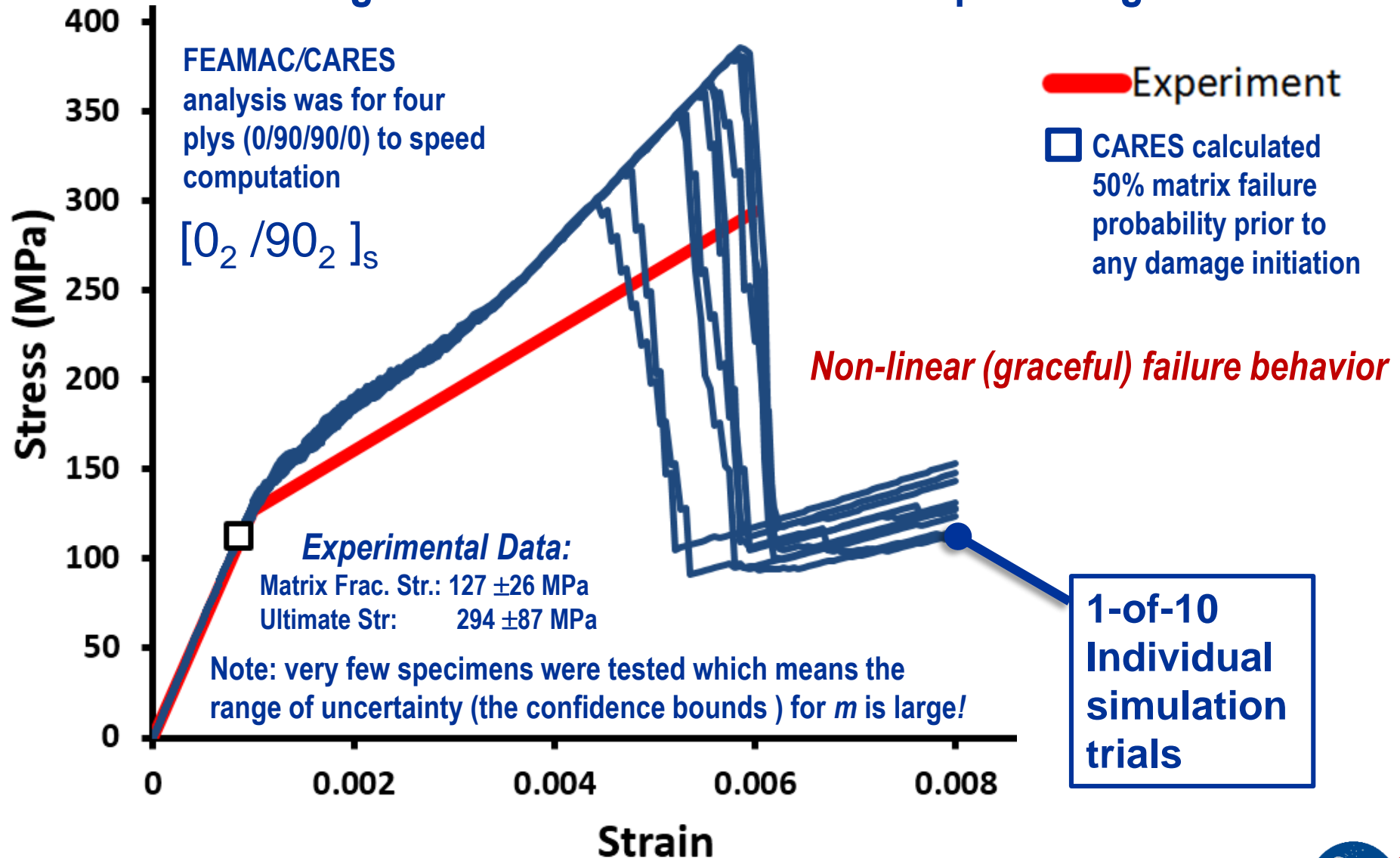
For $[+45_2/-45_2]_8$ Fiber Orientation; 10 trials at final (matrix) failure; deformed plots

FEAMAC/CARES analysis was for four plies (+45/-45/-45/+45) to speed computation

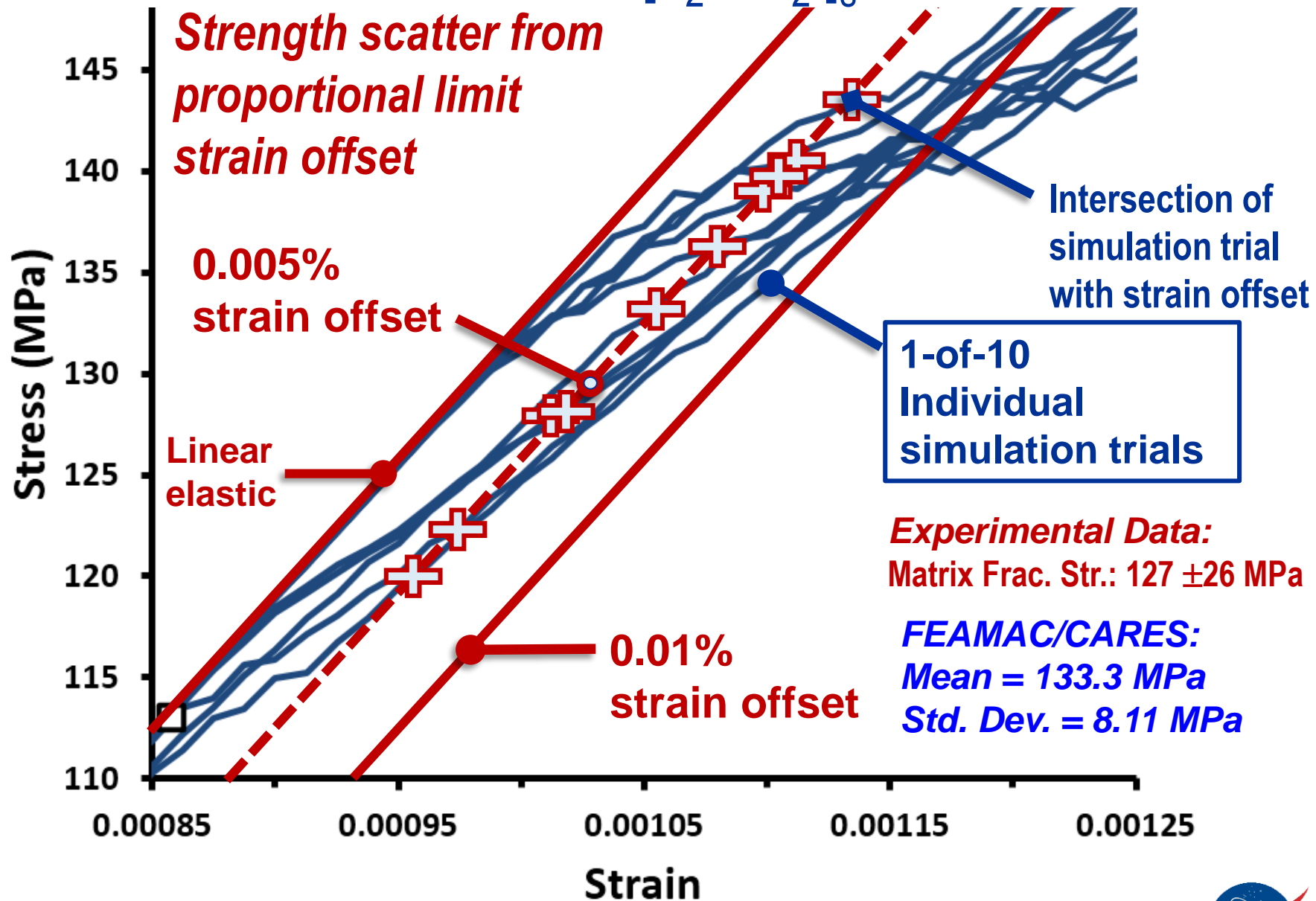


Prediction for Ten Trials for $[0_2/90_2]_s$ Fiber Orientation

Assuming matrix and interface are isotropic strength materials



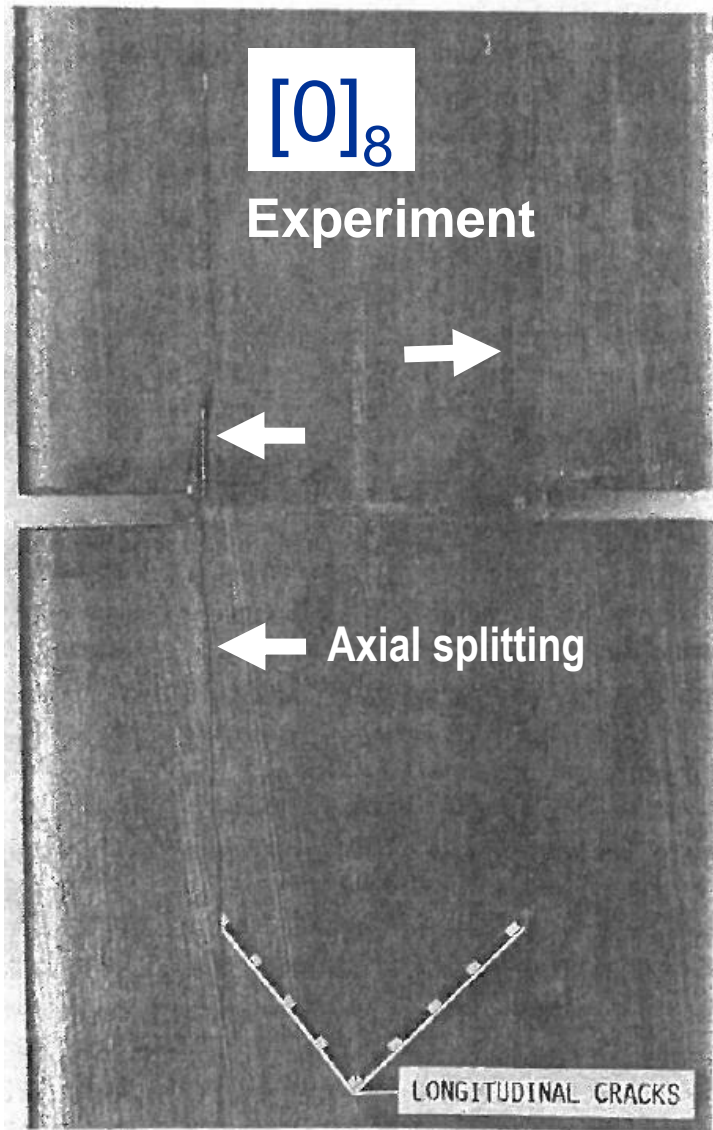
Prediction for Ten Trials for $[0_2/90_2]_s$ Fiber Orientation




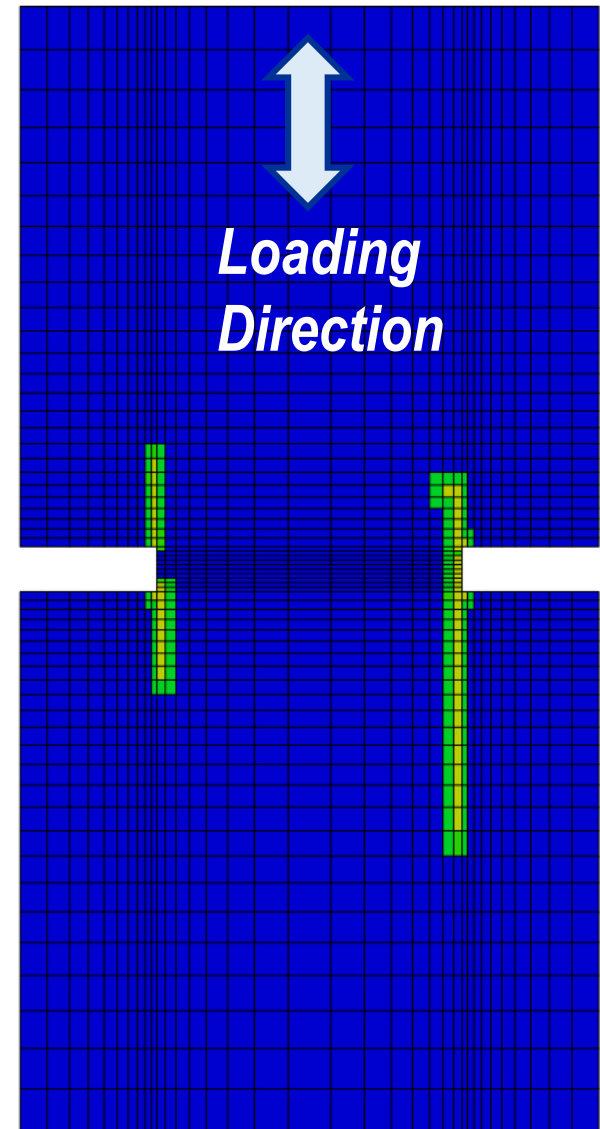
0° Double-Notched Tensile Specimen

Failure mode showed axial splitting of matrix

FEAMAC/CARES analysis was for a single ply to speed computation



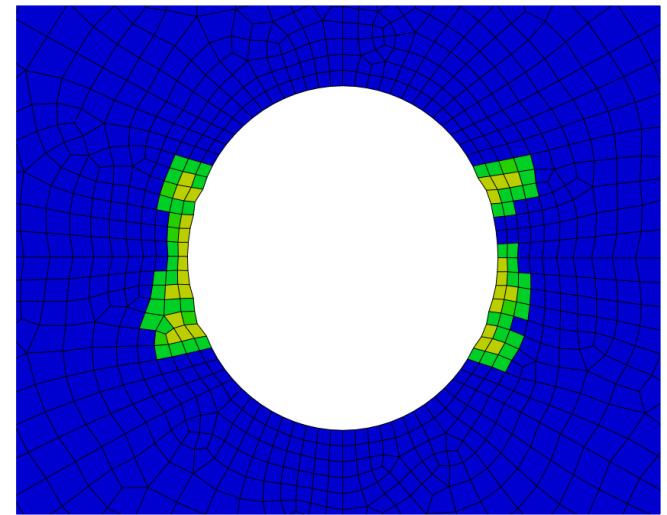
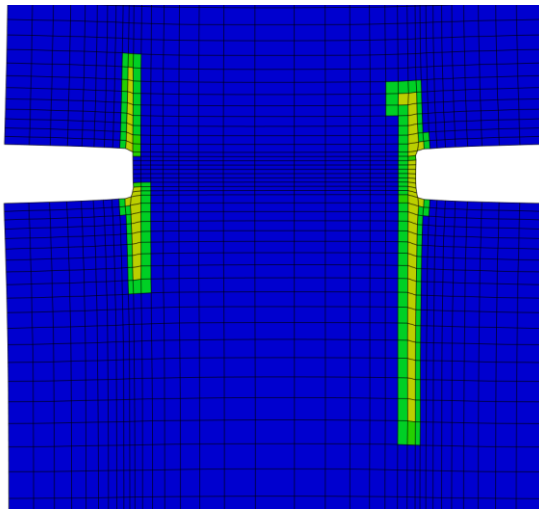

*Fiber
Direction*



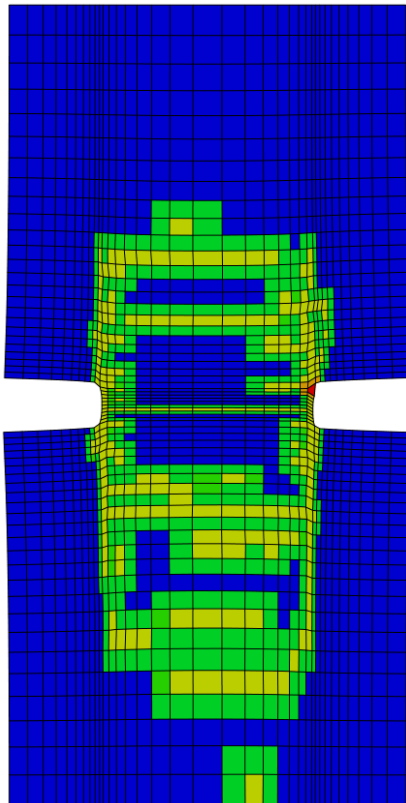
0° Double-Notched vs: Central-Hole Tensile Specimen



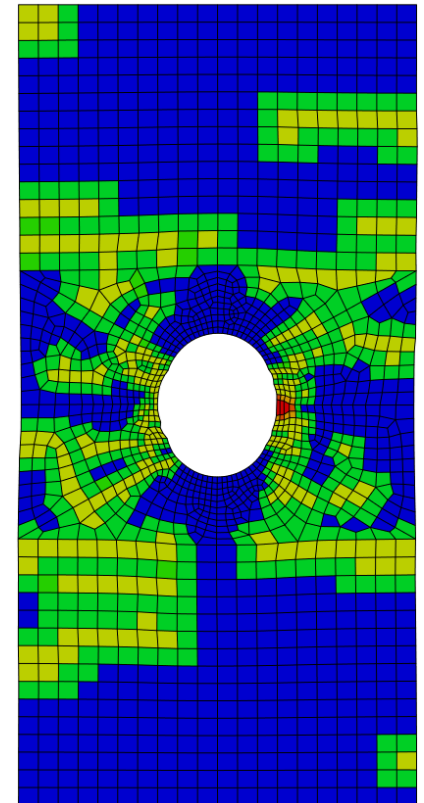
Loading Direction



Early matrix damage



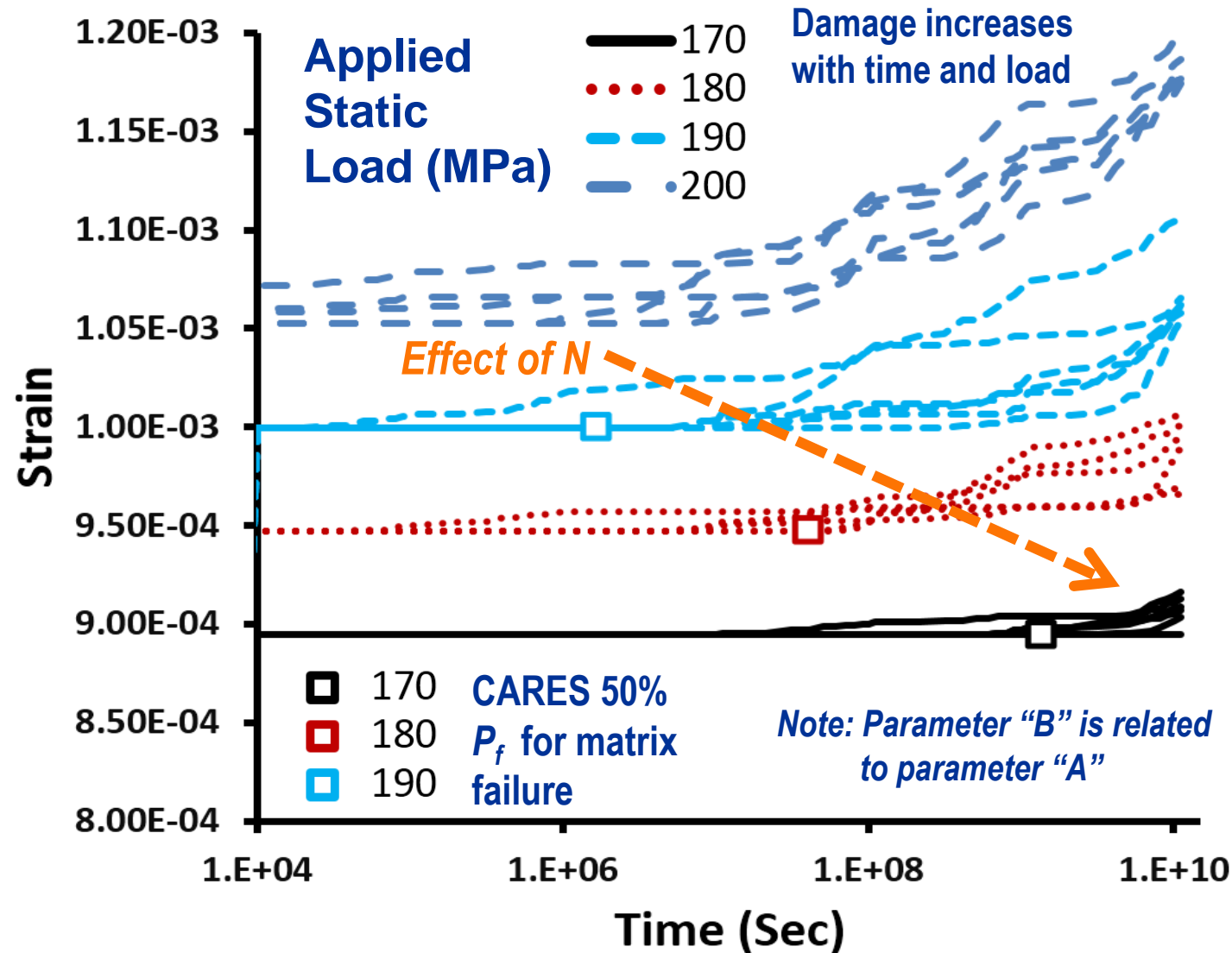
Matrix damage progression



Time-dependent Failure Example: Static Loading

(Matrix Damage Accumulation From Slow Crack Growth)

Strain response for applied static tensile load over time



Service life prediction

Longitudinal stress applied to a 0° SiC/RBSN ply

10 time increments per time magnitude

Slow Crack Growth Power Law:

$$\frac{da}{dt} = AK_{Ieq}^N$$

Weibull Parameters

$m = 7$ (Weibull slope)

$\sigma_0 = 106 \text{ MPa} \cdot \text{mm}^{3/7}$

Fatigue Parameters

$N = 20$ (fatigue slope)

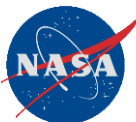
$B = 1.0E9 \text{ MPa}^2 \cdot \text{sec}$

Conclusions

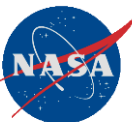
- Progressive damage simulation of composite structures incorporating probabilistic material strength models is possible with the FEAMAC/CARES code
- The Unit Sphere multiaxial model was used predict the strength response of a SiC-RBSN composite for various fiber orientations under uniaxial tension
- Reasonable correlation to matrix cracking strength experimental data was achieved assuming the matrix was an isotropic material with $m \approx 5$, and assuming residual stresses from thermal processing were present
- Brittle behavior vs: non-brittle failure (*graceful failure*) demonstrated
- Localized damage modes at stress concentration features shown

Acknowledgement

This work was funded by the NASA Transformative Tools and Technologies Program

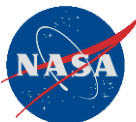


Extra Material

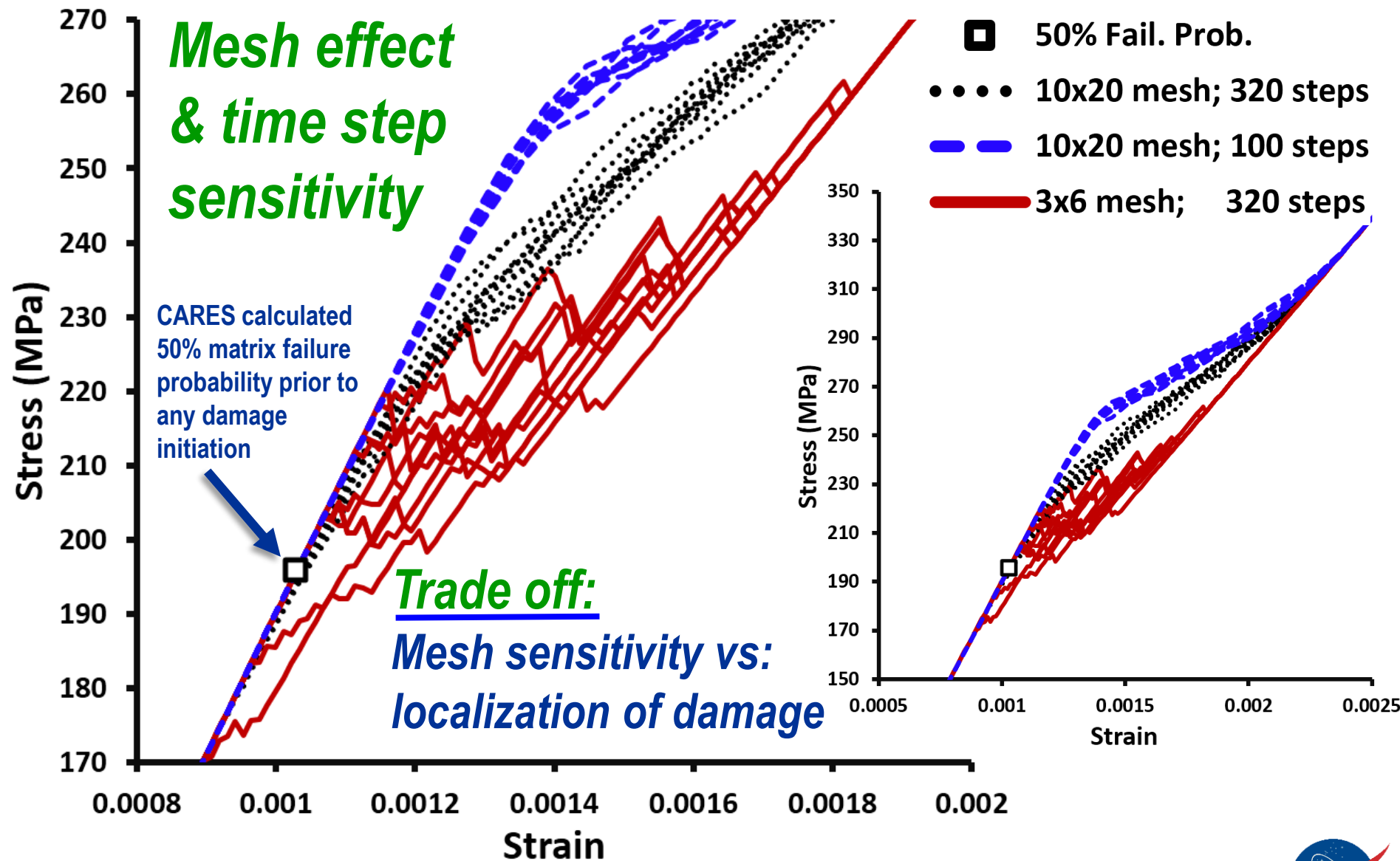


Abstract:

Reported here is a coupling of two NASA developed codes: CARES (Ceramics Analysis and Reliability Evaluation of Structures) with the MAC/GMC (Micromechanics Analysis Code/ Generalized Method of Cells) composite material analysis code. The resulting code is called FEAMAC/CARES and is constructed as an Abaqus finite element analysis UMAT (user defined material). Here we describe the FEAMAC/CARES code and an example problem (taken from the open literature) of a laminated CMC in off-axis loading is shown. FEAMAC/CARES performs stochastic-strength-based damage simulation response of a CMC under multiaxial loading using elastic stiffness reduction of the failed elements.



0° single ply tensile specimen *(Load parallel to fiber axis)*



Time-Dependent Life Prediction Theory - Slow Crack Growth and Cyclic Fatigue Crack Growth Laws

Power Law: - Slow Crack Growth (SCG)

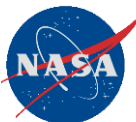
$$\frac{da}{dt} = A K_{Ieq}^N$$

Combined Power Law & Walker Law: SCG and Cyclic Fatigue

$$\frac{da}{dt} = A_1 g K_{Ieq}^N + A_2 f_c (1-R)^Q K_{Ieq}^N$$

Time-Dependent Life Prediction Theory - Slow Crack Growth and Cyclic Fatigue Crack Growth Laws with discrete time steps

Modeling individual time steps in the life prediction methodology enables simulating transient events such as turbine start-up/shut-down or atmospheric re-entry. A computationally efficient methodology has been developed that can extrapolate the reliability calculation for an arbitrary number of Z cycles – where each cycle is described by k number of time steps. This conceivably allows the coupling of other effects such as stiffness degradation and oxidation effects on the individual time steps and this can be accounted for interactively within the transient finite element and micromechanics analysis.



Transient Life Prediction Theory - Power Law SCG

Reliability formula for k discrete time steps over Z cycles:

$$P_{SV}(t_k) = \exp\left\{-\sum_{i=1}^n \frac{V_i}{4\pi} \int_{\Omega} \left[\dots \left[\left(\frac{\sigma_{Ieq,k,Tmax}}{\sigma_{0BV,k}} \right)^{N_{V,k}-2} + \right. \right. \right. \\ \left. \left. \left. \frac{\sigma_{Ieq,k}^{N_{V,k}} Z \Delta t_k}{\sigma_{0BV,k}^{N_{V,k}-2} B_{V,k}} \frac{m_{V,k}(N_{V,j}-2)}{m_{V,j}(N_{V,k}-2)} + \frac{\sigma_{Ieq,j}^{N_{V,j}} Z \Delta t_j}{\sigma_{0BV,j}^{N_{V,j}-2} B_{V,j}} \frac{m_{V,j}(N_{V,i}-2)}{m_{V,i}(N_{V,j}-2)} + \dots \right. \right. \\ \left. \left. \dots + \frac{\sigma_{Ieq,2}^{N_{V,2}} Z \Delta t_2}{\sigma_{0BV,2}^{N_{V,2}-2} B_{V,2}} \frac{m_{V,2}(N_{V,1}-2)}{m_{V,1}(N_{V,2}-2)} + \frac{\sigma_{Ieq,1}^{N_{V,1}} Z \Delta t_1}{\sigma_{0BV,1}^{N_{V,1}-2} B_{V,1}} \frac{m_{V,1}}{m_{V,1}(N_{V,1}-2)} d\Omega \right]_i \right\}$$

- **Individual time step:** Each time step can have different loading, Weibull, and fatigue parameters. Compatibility of failure probability is maintained between the individual time steps

SiC/RBSN Notional Example for SCG

0° Degree tensile specimen under a static load over time

- ❖ Use same 10x20 mesh, RUC, and material properties as previous SiC/RBSN off-axis loading example

Weibull and Slow Crack Growth (SCG) Parameters

Constituent	Weibull modulus, m_V	Weibull scale parameter, σ_{oV} , MPa • mm ^{3/m_V}	Fatigue exponent, N_V (Equation (11))	Fatigue constant, B_V , MPa ² • sec (Equation (17))
Fiber	20.0	2875.0	100.0	1.0×10 ¹⁰
Matrix	7.0	106.0	20.0	1.0×10 ⁹
Interface	7.0	60.0	100.0	1.0×10 ¹⁰

Unit Sphere Multiaxial (Batdorf) Model:

Puts linear elastic fracture mechanics into Weibull weakest-link theory

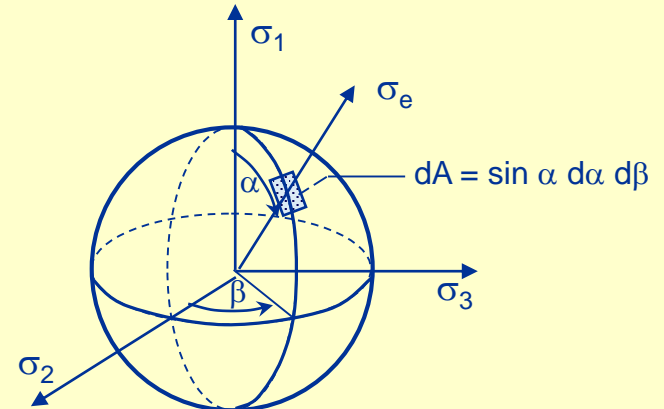
- **Incremental failure probability is the product of two probabilities:**

$$\Delta P_f = P_1 \cdot P_2$$

P_1 = Probability of the existence of a crack having a critical strength between σ_c and $\sigma_c + \Delta\sigma_c$ in the incremental volume ΔV

P_2 = Probability a crack having a critical strength of σ_c will be oriented in a direction such that it will fail under the applied multiaxial stress state

P_2 involves Integration of an equivalent stress σ_e , where $\sigma_e \geq \sigma_c$, over the surface of a unit radius sphere (all possible flaw orientations) divided by the total surface area of the unit radius sphere



σ_e is a function of an assumed crack shape and multiaxial fracture criterion

Mixed-Mode Fracture Criteria:

- Normal stress (shear-insensitive cracks)
- Maximum tensile stress
- Total coplanar strain energy release rate
- Noncoplanar (Shetty)

Flaw Shapes:

- Griffith crack
- Penny-shaped crack

- **Component failure probability:**

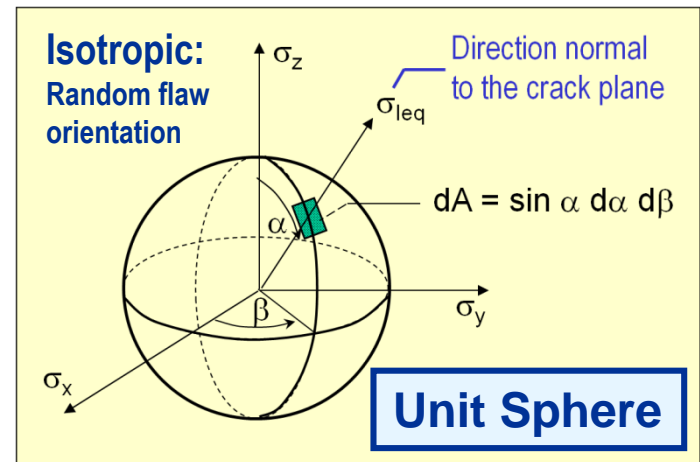
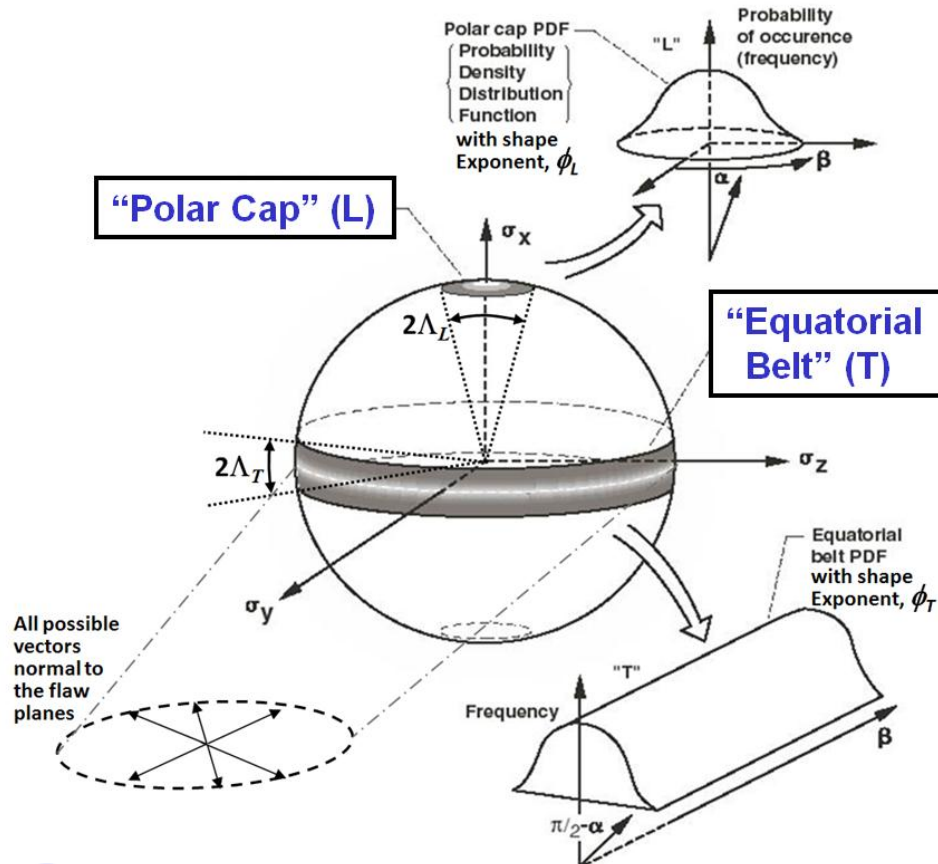
$$P_f = 1 - \exp \left\{ - \int_V \left[\int_0^{\sigma_e} P_1(\sigma_c) P_2(\sigma_c) d\sigma_c \right] dV \right\}$$

CARES Unit Sphere Multiaxial model

has crack geometry & mixed-mode fracture criterion

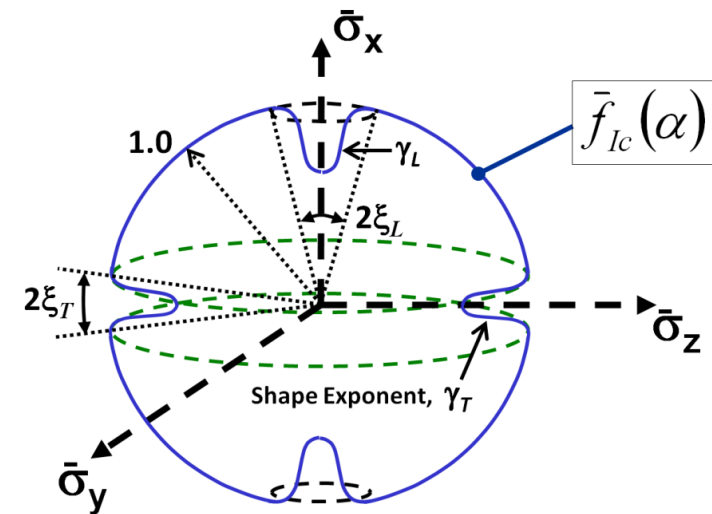
➤ Two models for transverse isotropy

(1) Flaw / Fracture-Plane Orientation Anisotropy



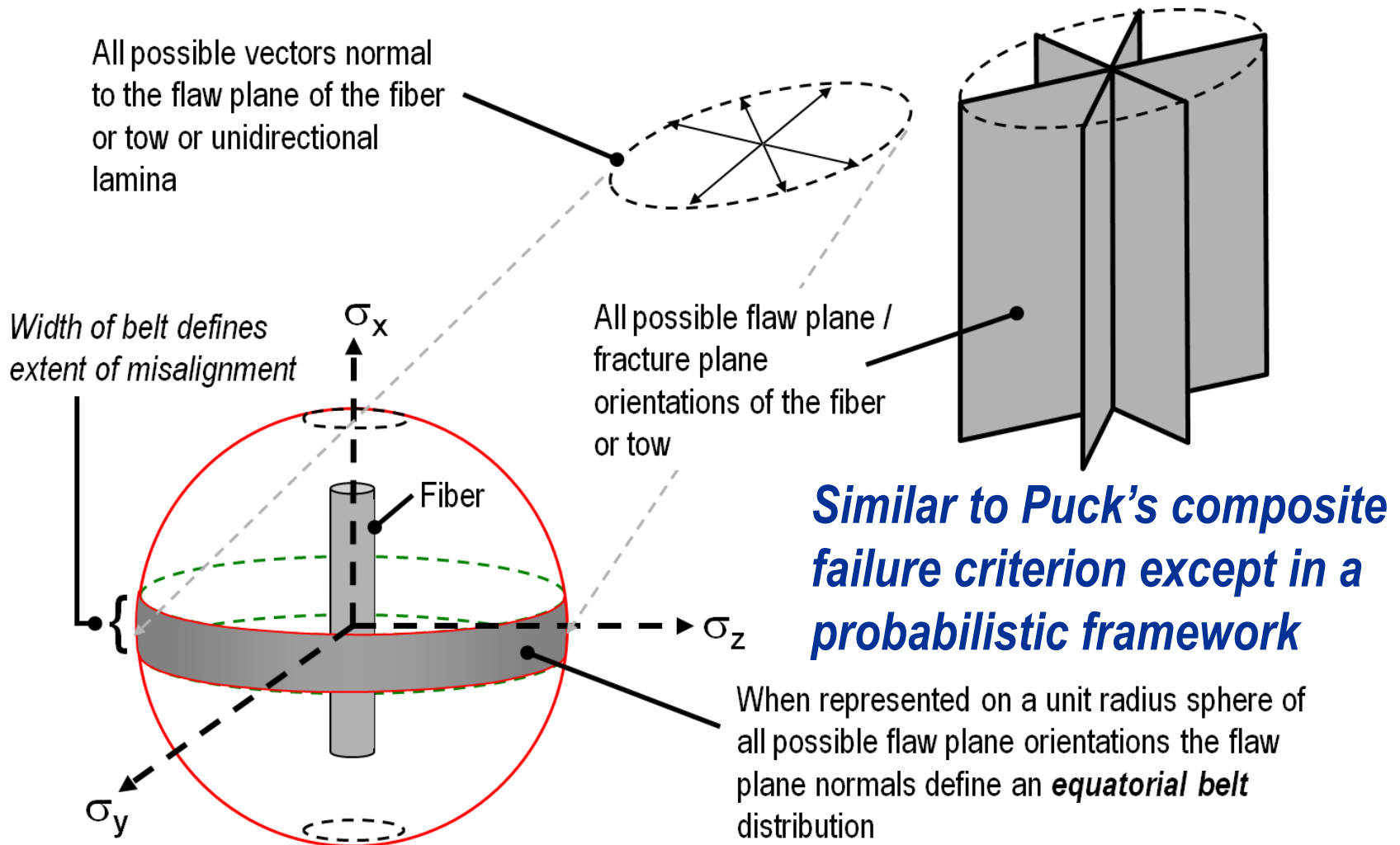
(2) Strength Orientation Anisotropy

σ_{lc} or K_{lc} varies with orientation



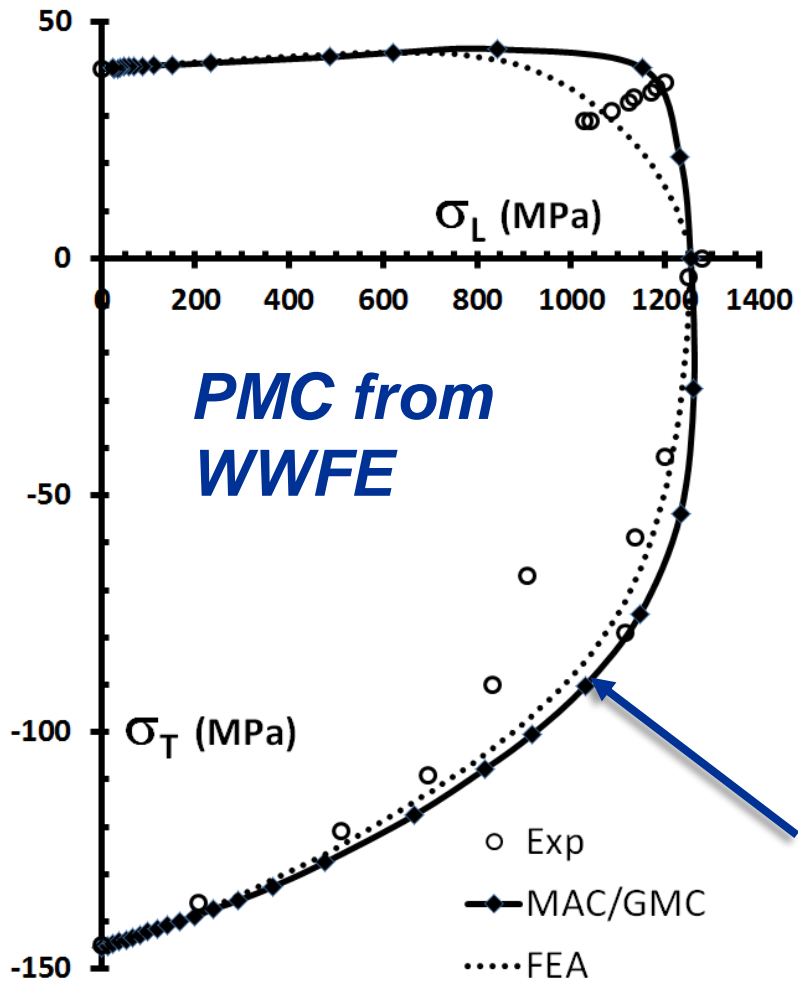
Nemeth, N.N. (2014): Unit-sphere multiaxial stochastic-strength model applied to a composite material. J. Comp. Mat., Vol. 48(27) Nov. 2014, pp. 3395-3424.

Anisotropic Unit Sphere model defined in a material coordinate system reference frame

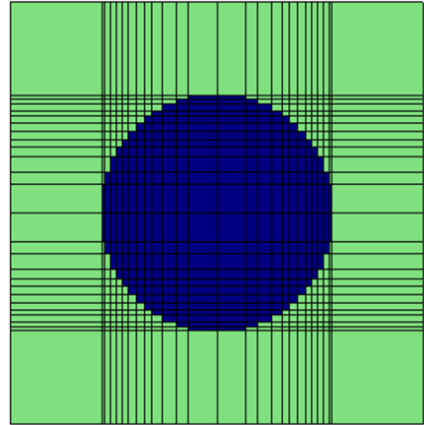


Multiaxial Performance: biaxial response predicted from a MAC/GMC RUC for combined longitudinal (L) and transverse (T) loading on a unidirectional PMC vs: FEA.

50% probability of failure envelope.



GMC RUC Used

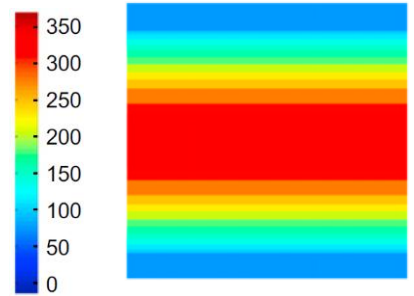


Unit Sphere parameters adjusted so GMC results matched FEA results for uniaxial tension and compression.

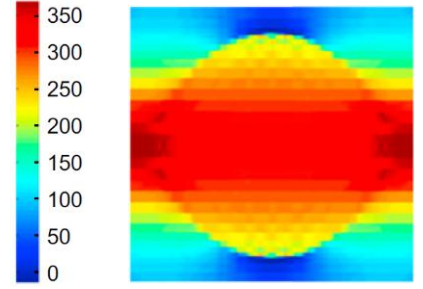
Intermediate points are predictions

Differences in RUC stress fields for a transverse strain:

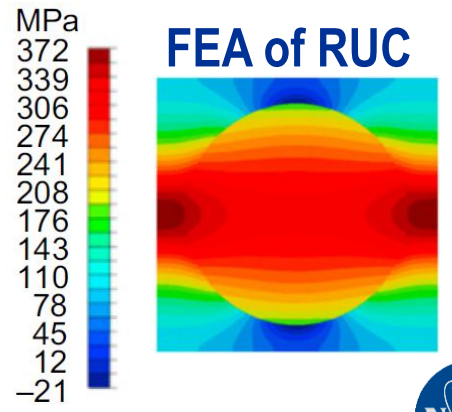
GMC RUC



HF - GMC RUC

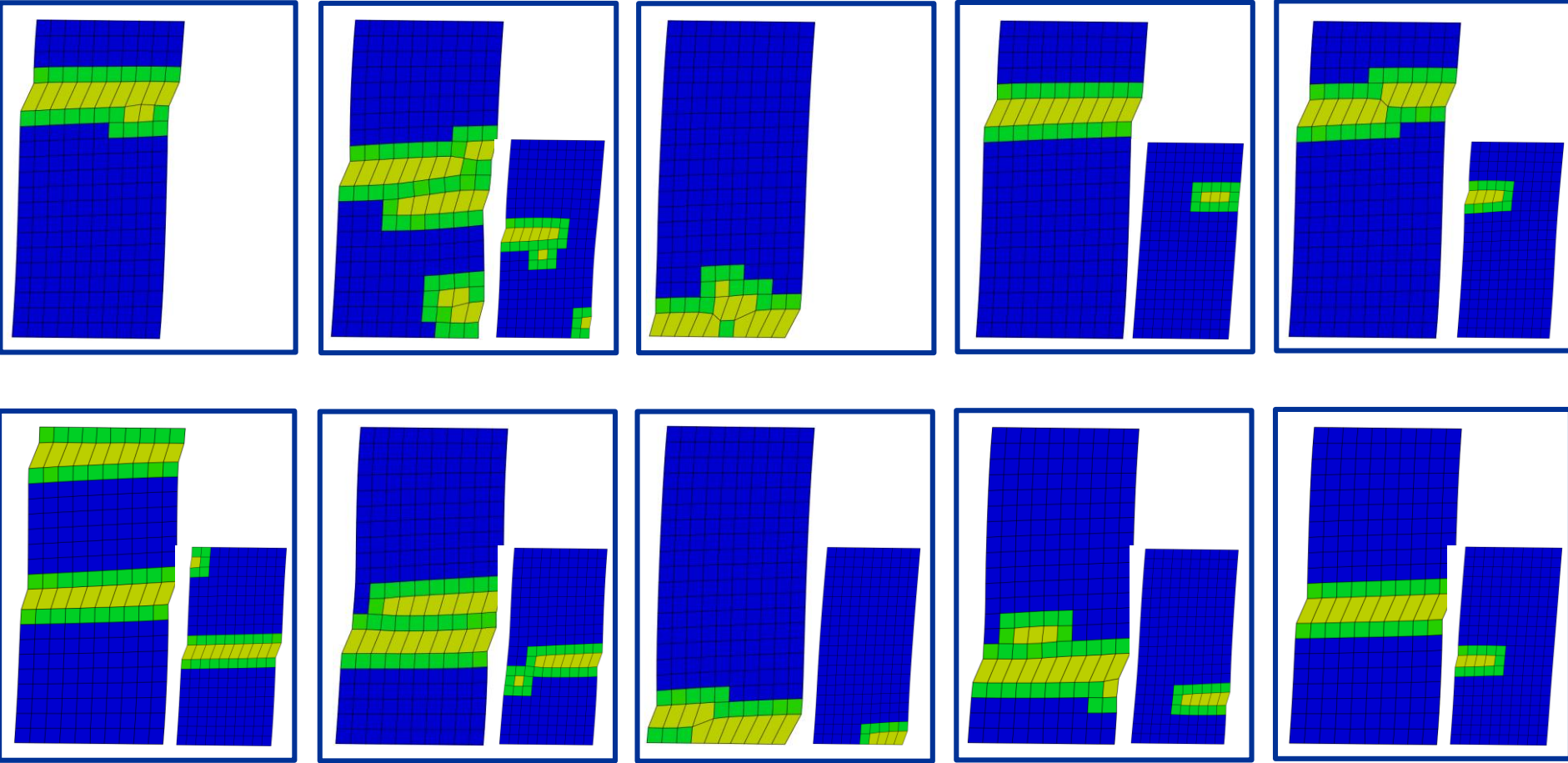


FEA of RUC



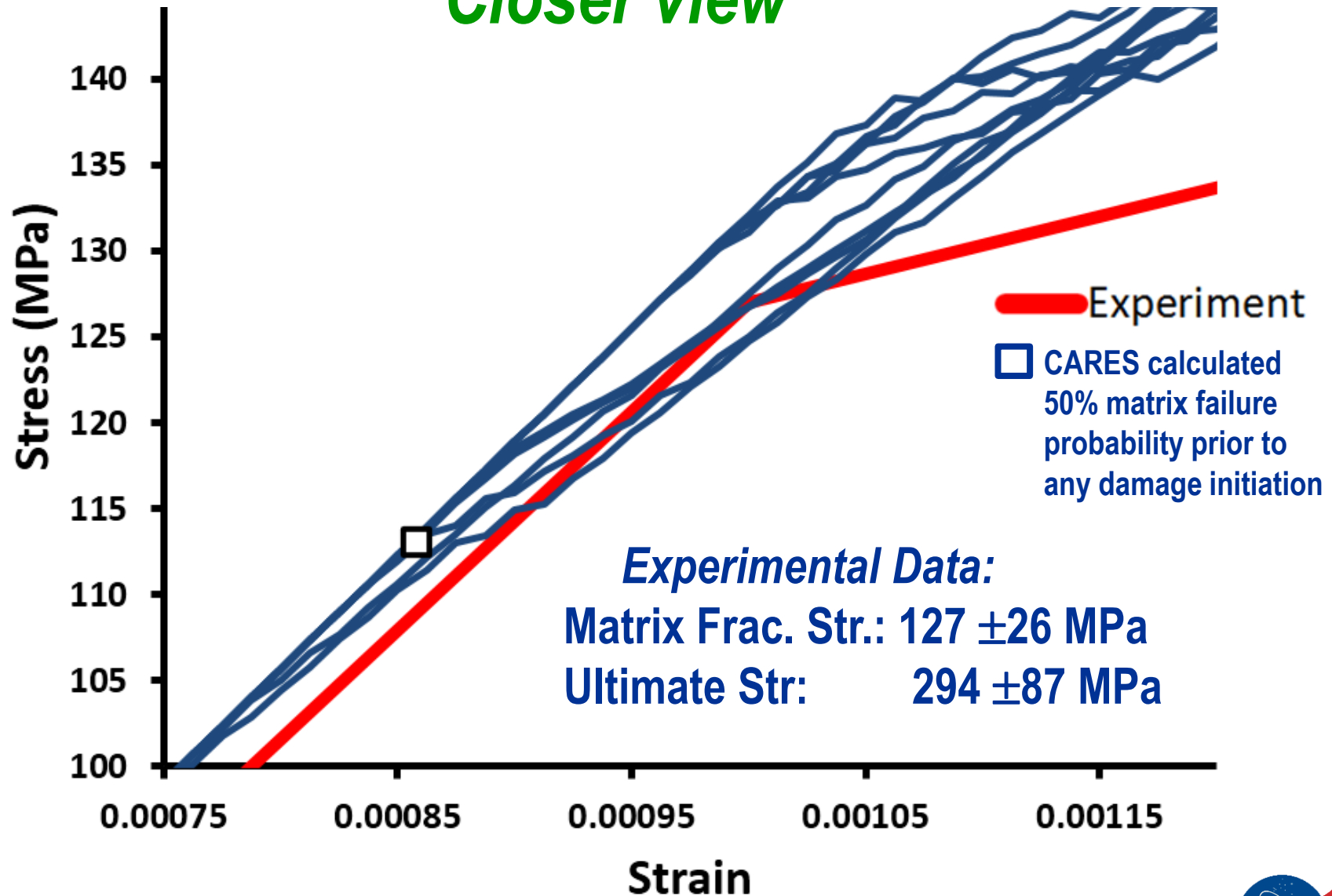
45° off-axis tensile specimen; 10 trials at final failure deformed plots

- Edges are allowed to freely deform (warp) on cool-down
- *After cool-down*; bottom edge fixed in loading direction when displacement load applied
- *After cool-down*; single node along top edge (middle) fixed in direction perpendicular to displacement direct.



Prediction for Ten Trials for $[0_2/90_2]_s$ Fiber Orientation

Closer view

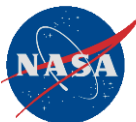
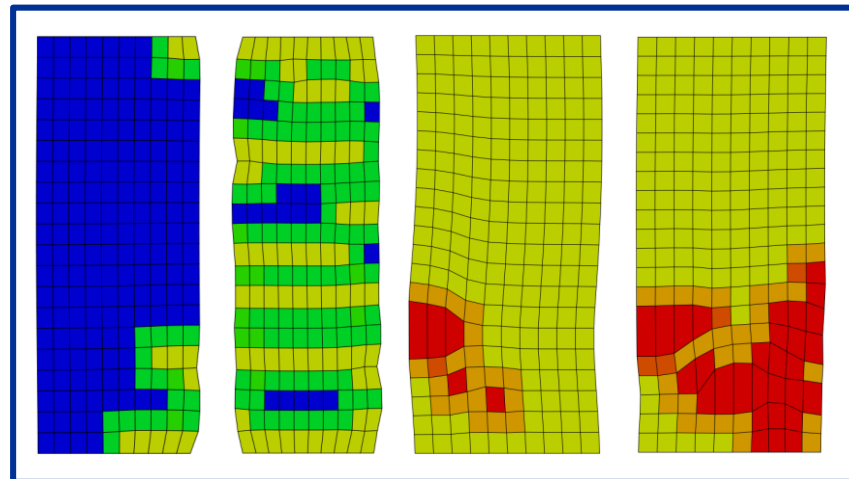
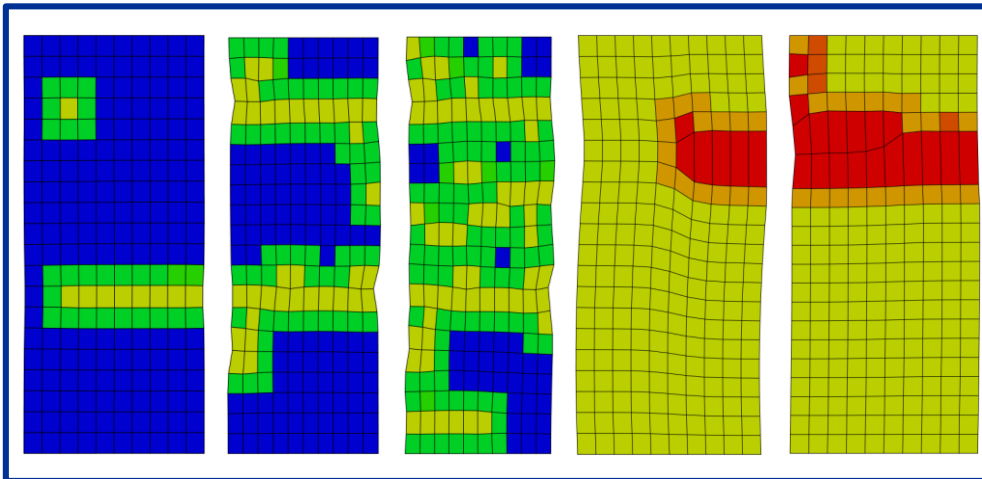
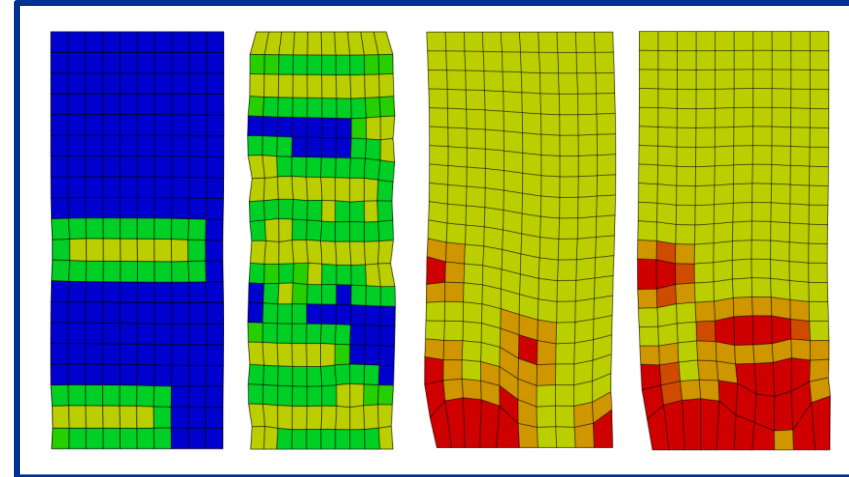
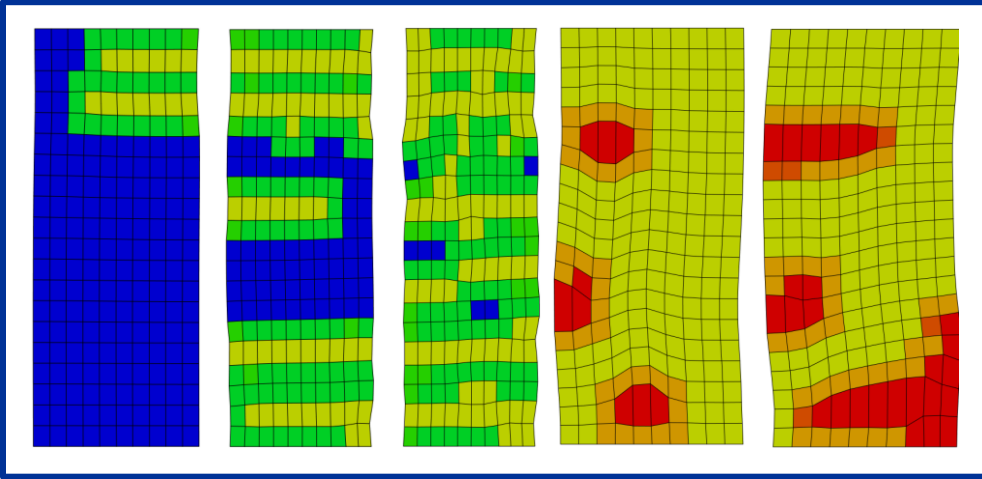


For $[0_2/90_2]_s$ fiber orientation; four trials with deformed plots

Progression from matrix failure to final fiber failure

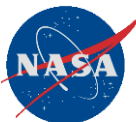


FEAMAC/CARES analysis was for four
plys (0/90/90/0) to speed computation



Path Forward

- Continue demonstrate/benchmark capability on CMCs *(using available literature)*
 - **For uniaxial & multiaxial failure response**
(orientation, lamination, stress concentration, flexural)
 - Fast-fracture
 - Time & cycle dependent
 - more detailed micromechanical models of failure modes
- Develop / incorporate enviromechanical degradation models
- Investigate applicability to predict EBC damage progression
- Develop / incorporate anisotropic elastic modulus degradation based on CARES critical fracture angle probability density distribution
- Improve software efficiency (memory, speed, multiprocessing)
- ❖ **Demonstrate this capability on component/structure**



Approach For Life Prediction & Component Design Of Composites

- **Combine CARES, MAC & FEA codes** where
Micromechanics provides the link between structures & materials

FEAMAC/CARES
(Abaqus UMAT)

CARES: *monolithic ceramics*

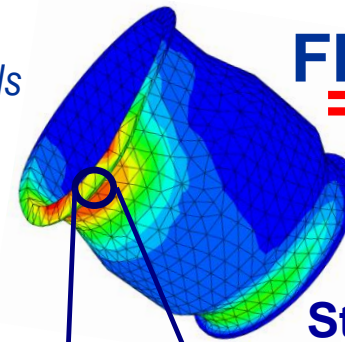
- Probabilistic strength
- Mechanistic-based multiaxial failure model
- Efficient life prediction algorithm
- Isotropic and *transverse isotropy*

New

MAC/GMC: *composites analysis*

- Micromechanics
- Accurate RUC stress fields
- Flexibility in RUC designs
- Progressive damage capability
- Computationally efficient

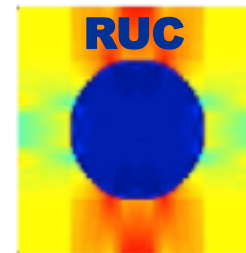
Move CARES from the macroscopic scale of the structure to the microscale of the individual material constituents & RUC with the FEA-MAC micromechanics code



Structural-Scale FEA



Element/Integration Point



Micromechanics Analysis

(MAC/GMC)

(CARES)
Subroutine

Reliability analysis at the RUC level



Fiber Interface Matrix

